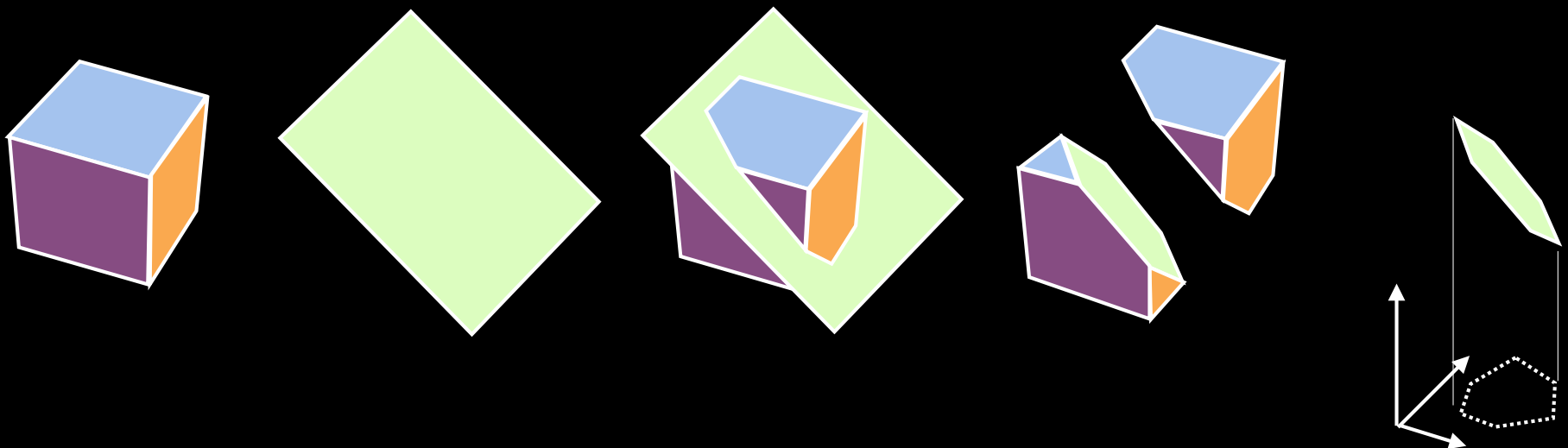
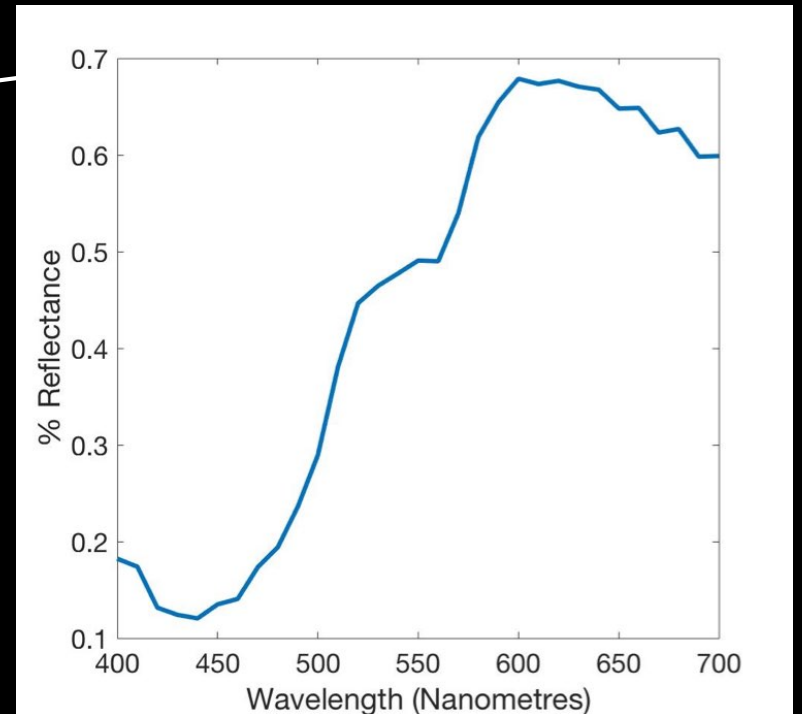


Metamer Sets

Graham D. Finlayson
University of East Anglia



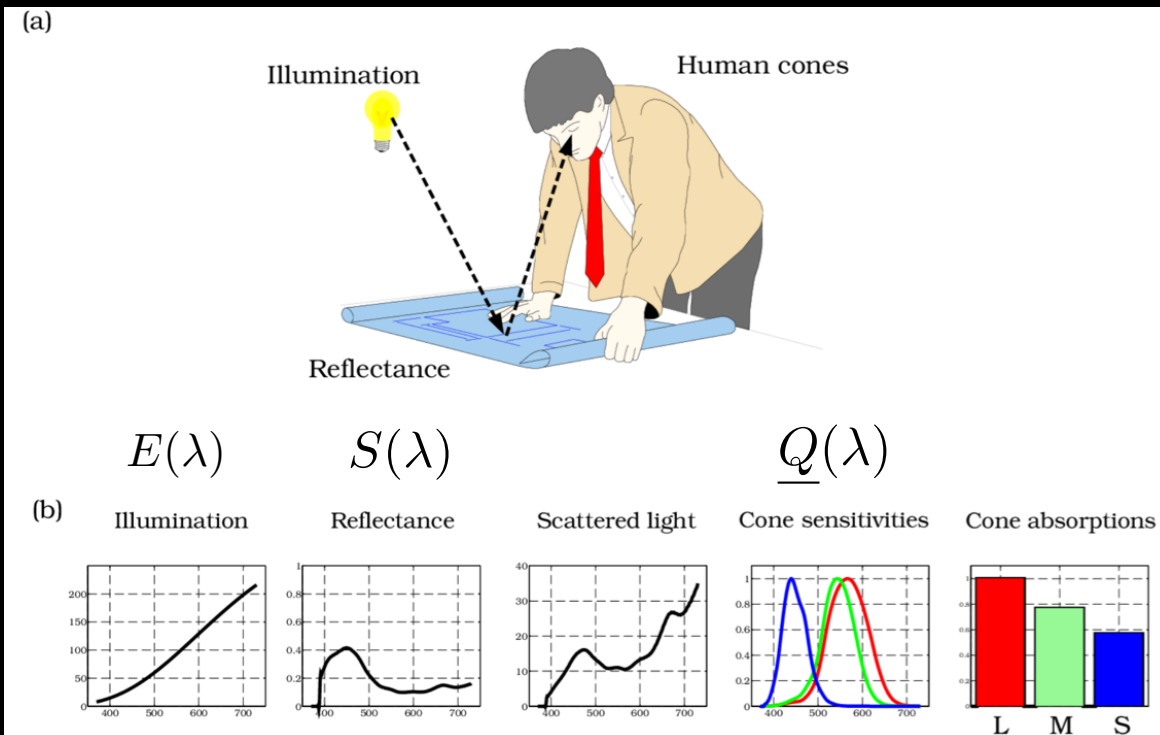
RGB to Spectra



Nitre Challenge

Metamerism

Simple Image Formation



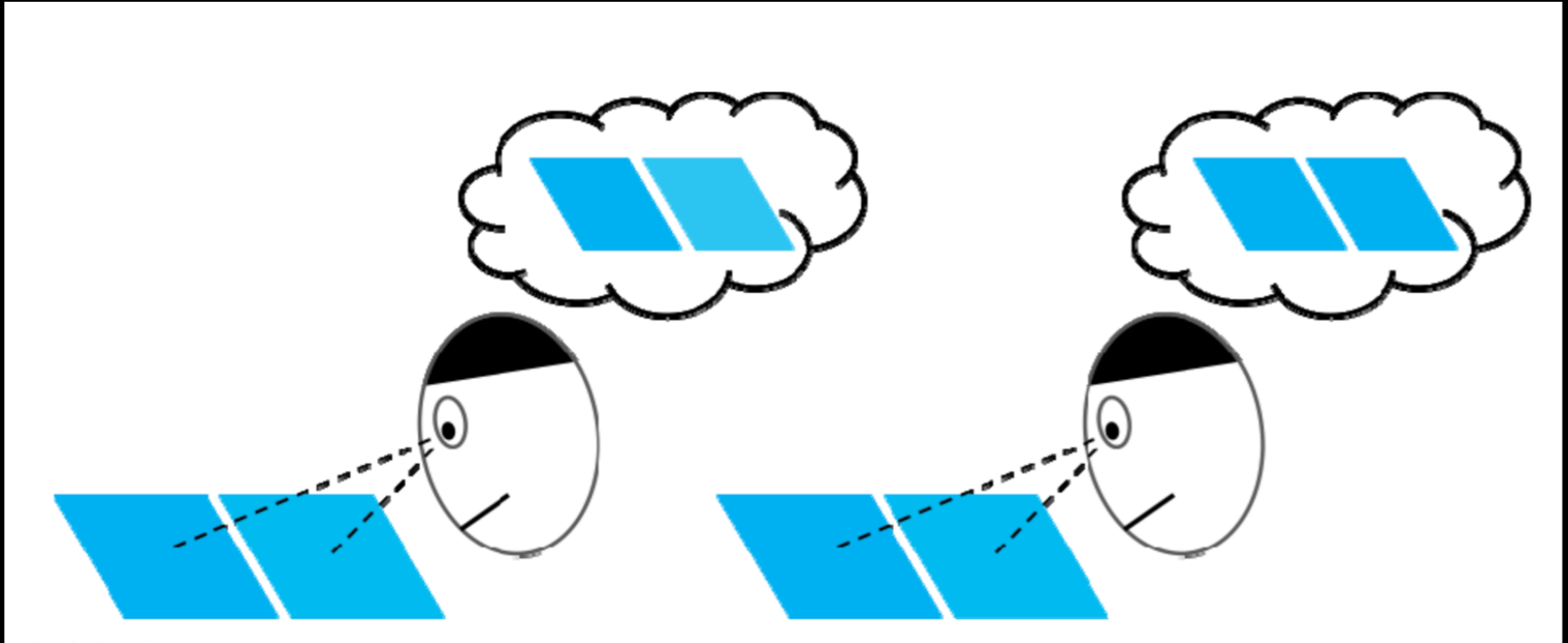
$$\int_{\omega} \underline{Q}(\lambda) E(\lambda) S(\lambda) d\lambda$$

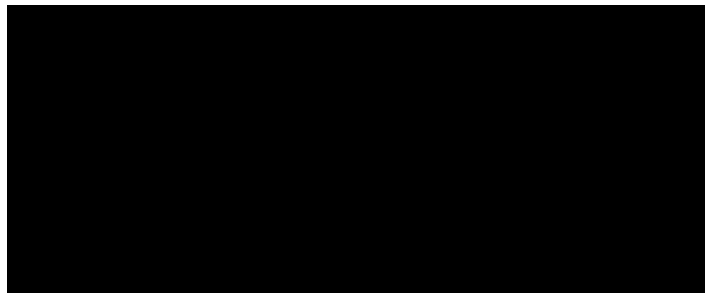
$$\underline{R}(\lambda) = \underline{Q}(\lambda) E(\lambda)$$

$$\int_{\omega} \underline{R}(\lambda) S(\lambda) d\lambda$$

Wandell, BA. "Foundations of Vision"

Observer Metamerism





Emissive chart for imager calibration

*Jeffrey M. DiCarlo, Glen Eric Montgomery and Steven W. Trovinger
Hewlett-Packard Laboratories
Palo Alto, California*

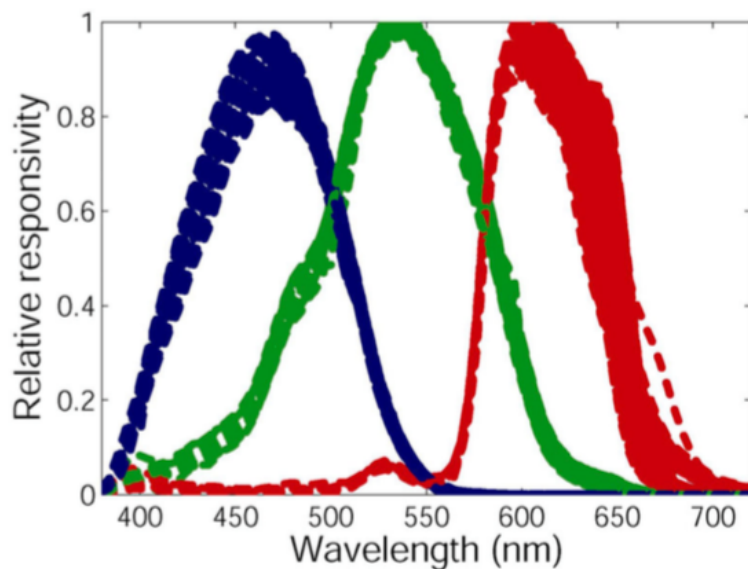


Figure 1. Responsivity functions for two hundred instances of a consumer camera plotted on top of one another. The widths of the lines indicate the responsivity variations across camera instances.



Emissive chart for imager calibration

*Jeffrey M. DiCarlo, Glen Eric Montgomery and Steven W. Trovinger
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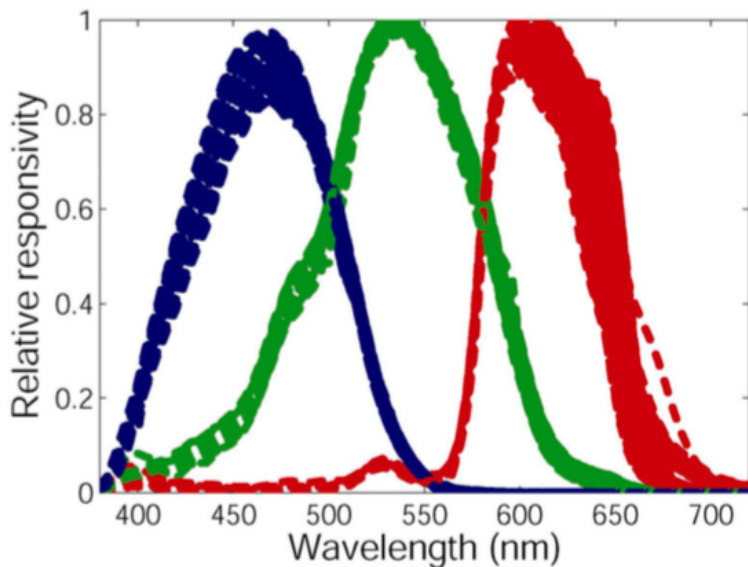


Figure 1. Responsivity functions for two hundred instances of a consumer camera plotted on top of one another. The widths of the lines indicate the responsivity variations across camera instances.



Figure 6. The first prototype of the emissive calibration chart.

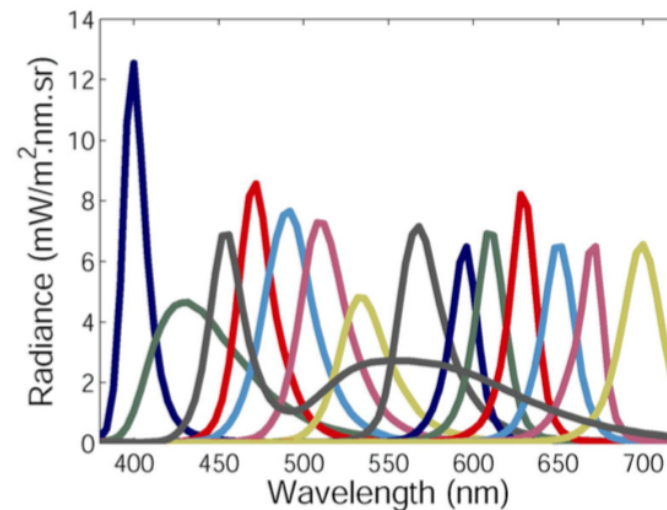
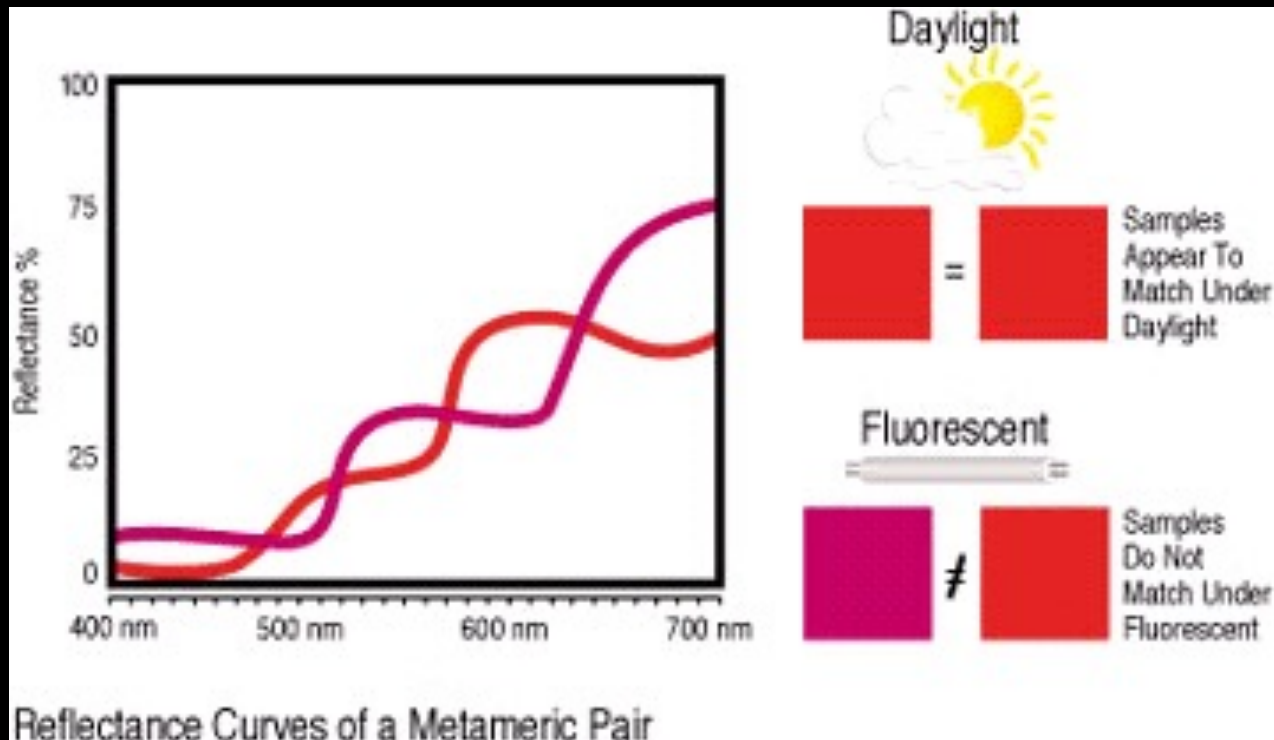


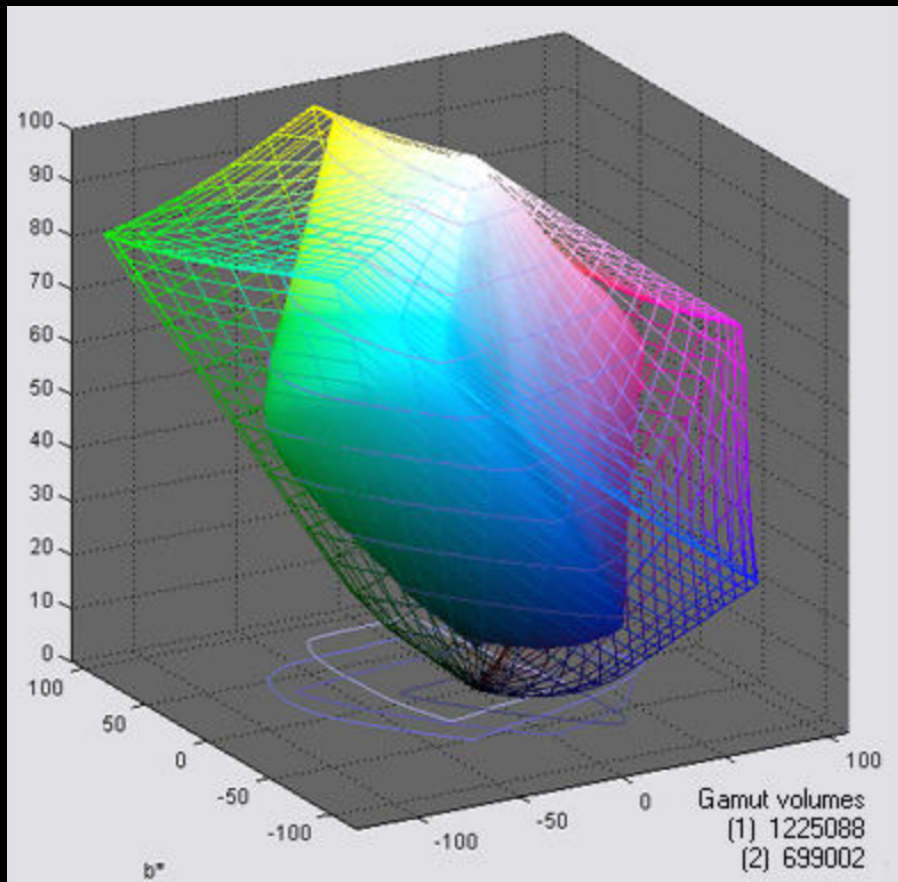
Figure 7. The spectral power distributions of the LED light sources used in the emissive calibration chart.

Illuminant Metamerism



Idea of Gamuts:

A stepping stone to thinking about metamers

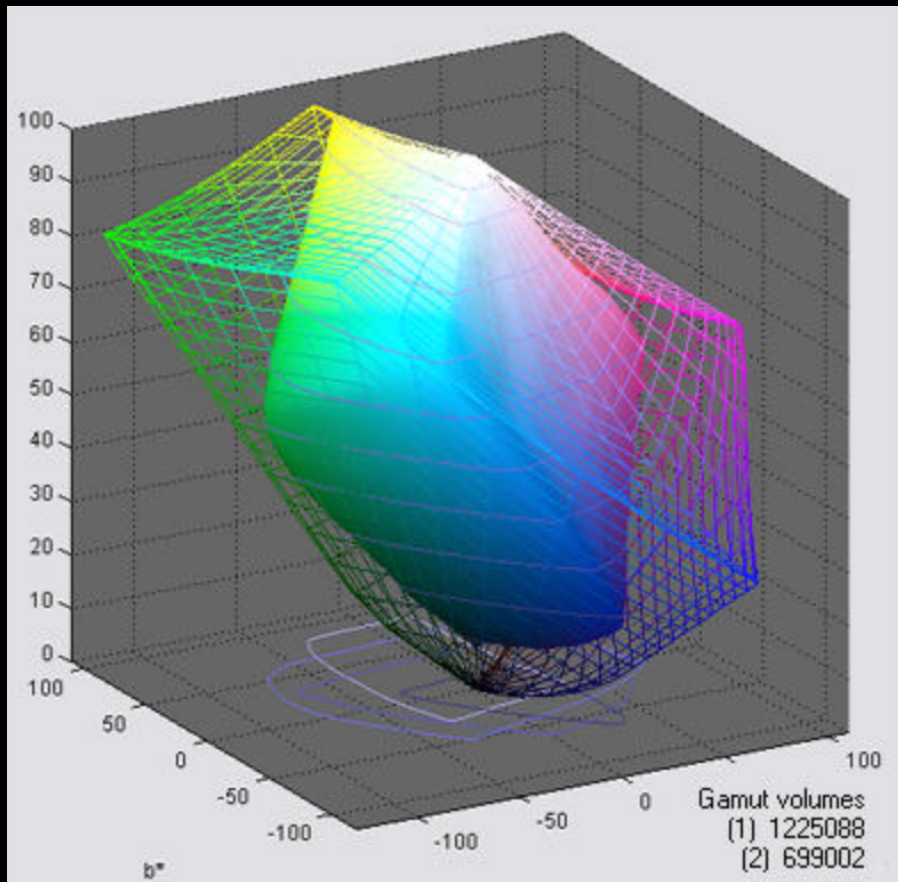


Wire Frame: Adobe RGB

Solid: Epson Printer

Idea of Gamuts:

A stepping stone to thinking about metamers



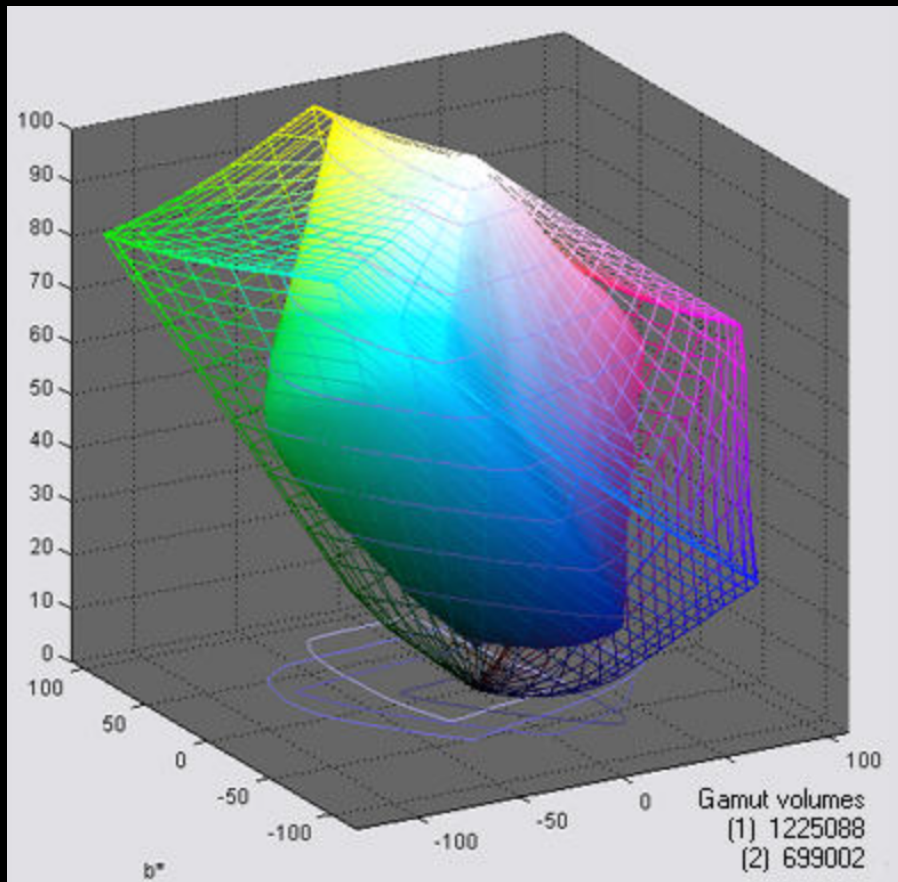
Wire Frame: Adobe RGB

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i) If we could manufacture any reflectance (theoretically possible), what would be the gamut of colours?

Idea of Gamuts:

A stepping stone to thinking about metamers



Wire Frame: Adobe RGB

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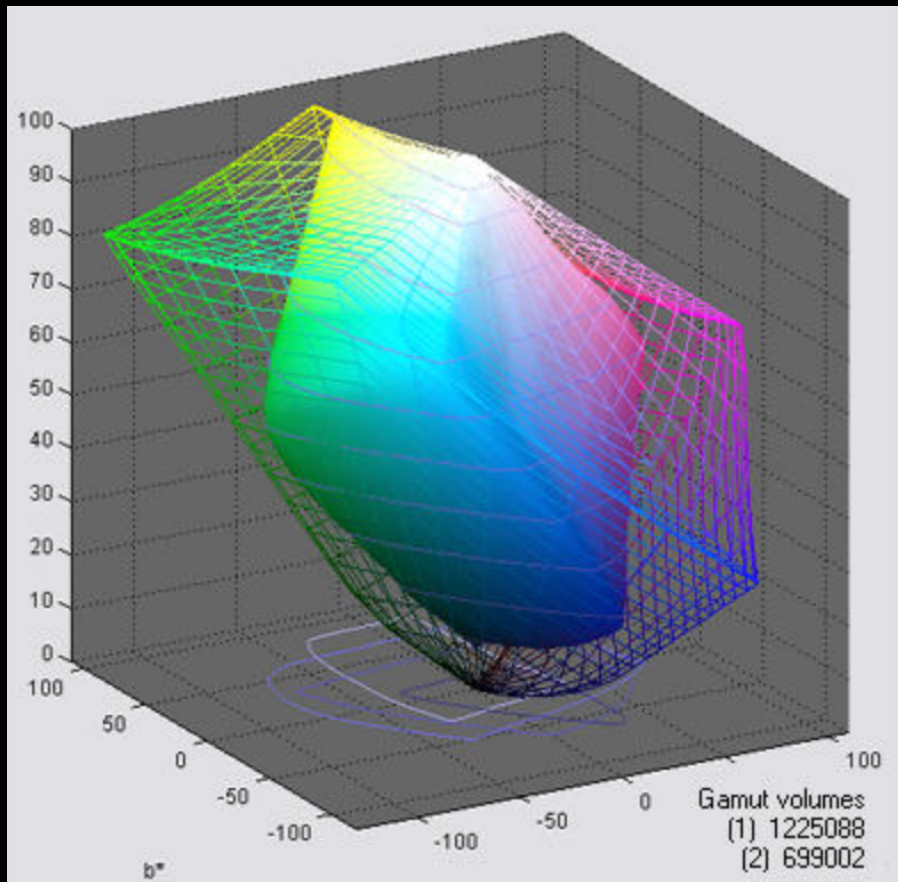
i) If we could manufacture any reflectance (theoretically possible), what would be the gamut of colours?

ii) This theoretical gamut is called the Object Colour Solid (OCS)

How the OCS is computed (efficiently) has been the source of research

Idea of Gamuts:

A stepping stone to thinking about metamers



Wire Frame: Adobe RGB

Solid: Epson Printer

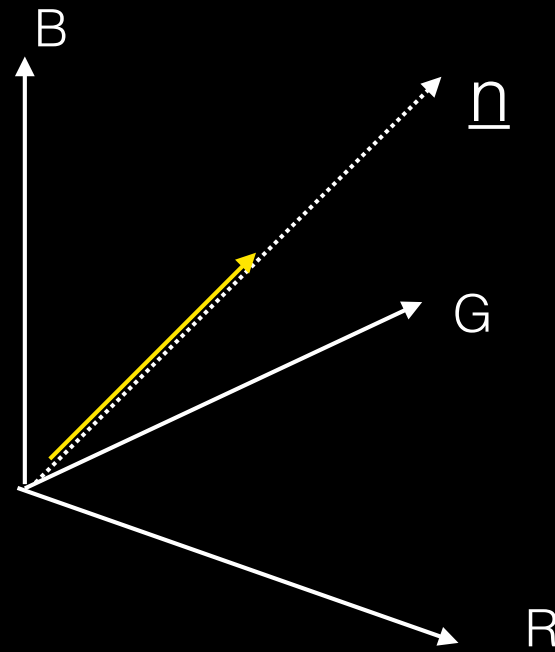
i) If we could manufacture any reflectance (theoretically possible), what would be the gamut of colours?

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iii) How we solve for the OCS can help us solve for pairs of reflectances that are metamers

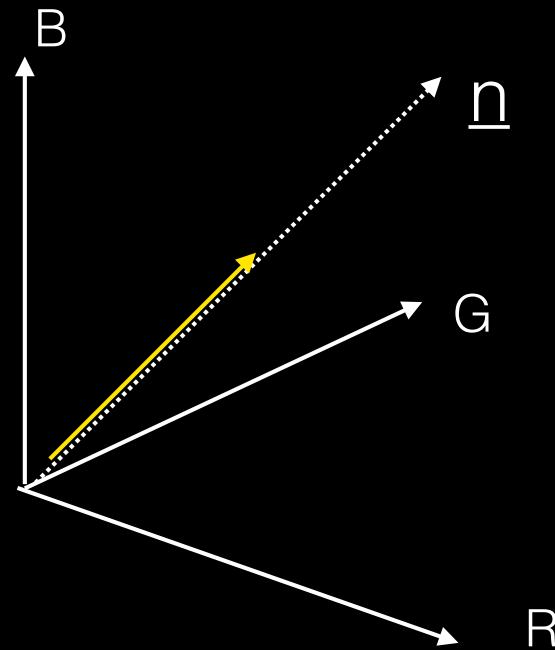
Theoretical limits: Object Colour Solid



What is the colour response \underline{p} in the direction \underline{n} which has maximum length?

$$\max_{S(\lambda)} ||\underline{n} \cdot \underline{p}|| \quad s.t. \quad \begin{cases} 0 \leq S(\lambda) \leq 1 \\ \underline{p} = \int_{\omega} \underline{R}(\lambda) S(\lambda) d\lambda \\ (\underline{I}_{3 \times 3} - \underline{n} \underline{n}^t) \underline{p} = \underline{0} \end{cases}$$

Theoretical limits: Object Colour Solid

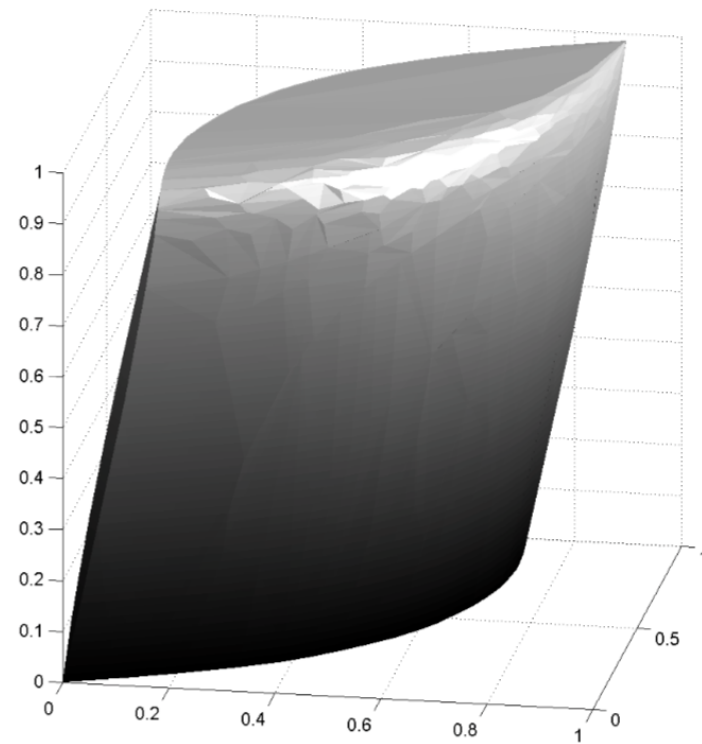
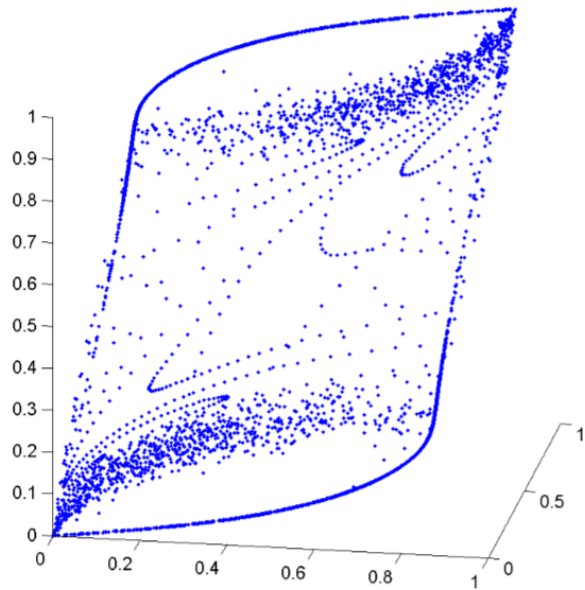


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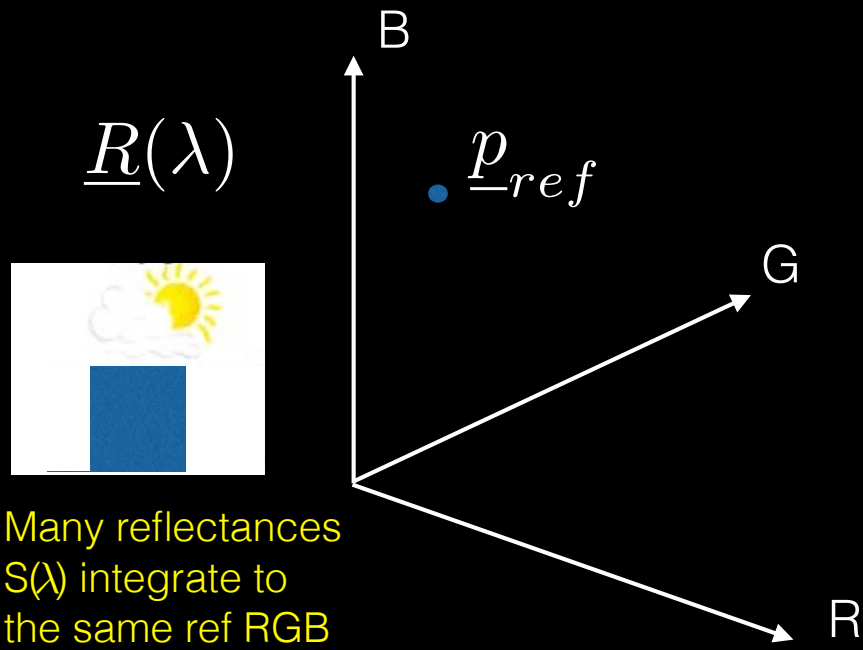
In the discrete domain, this optimisation is a 'Quadratic Program'. Can be solved Efficiently. (Actually, can be further simplified as a linear program)

Object Colour Solid



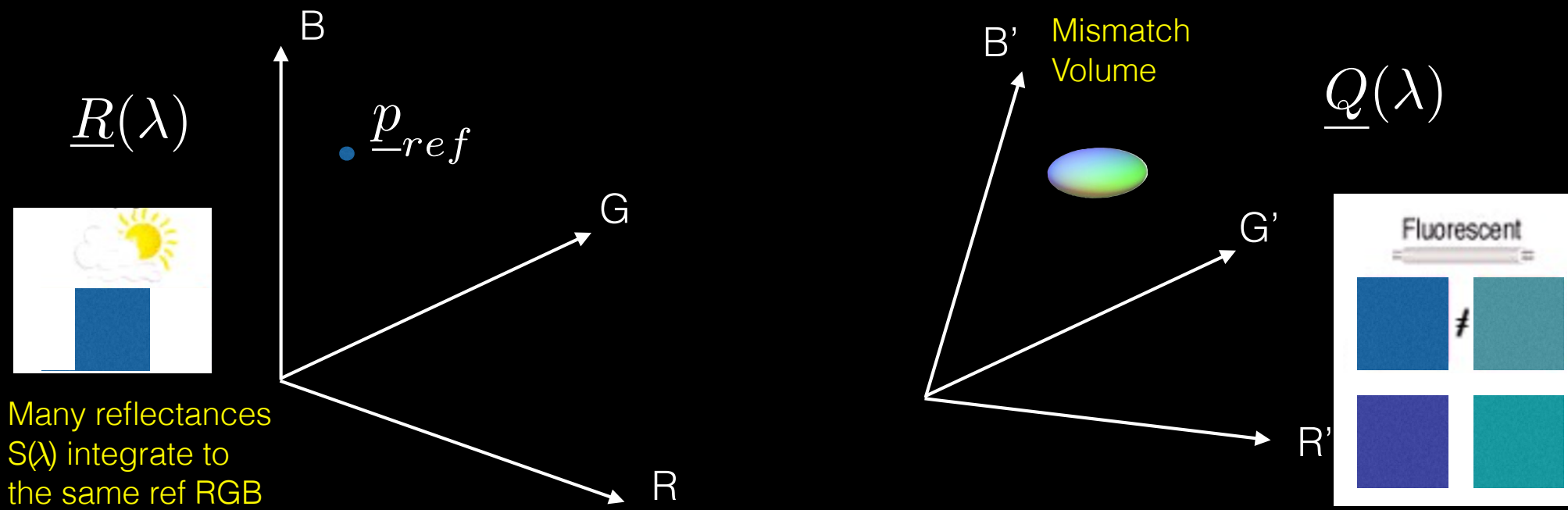
Metamer Mismatch Volume

a measure of how well spectrum is recovered



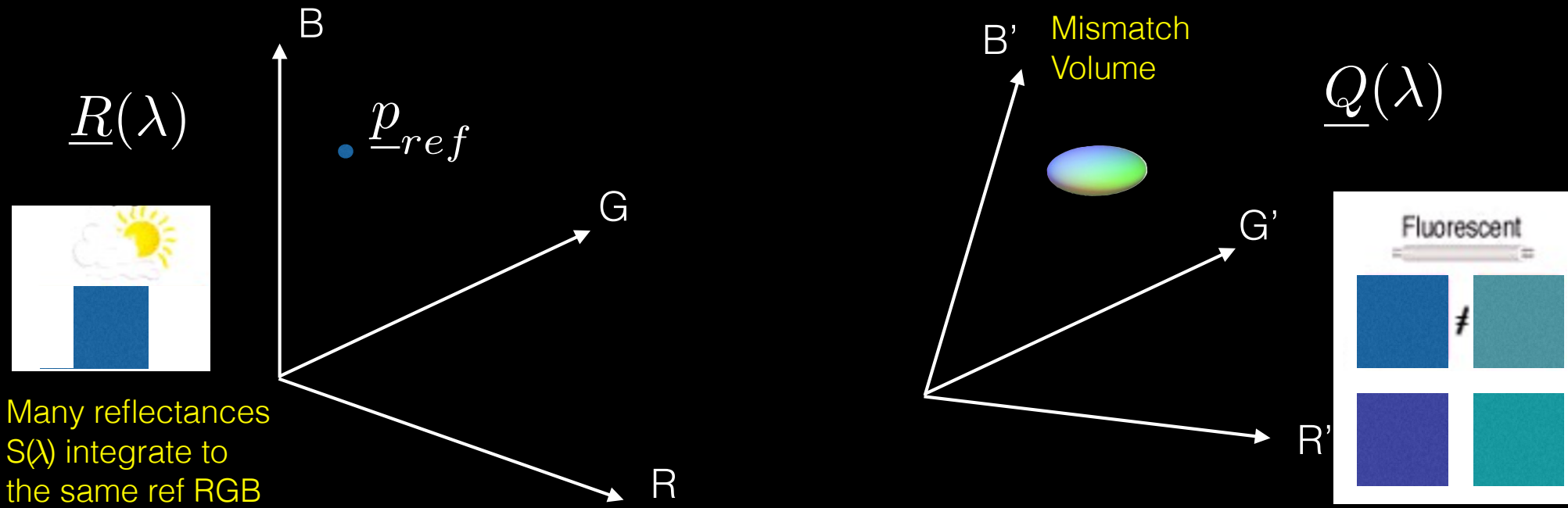
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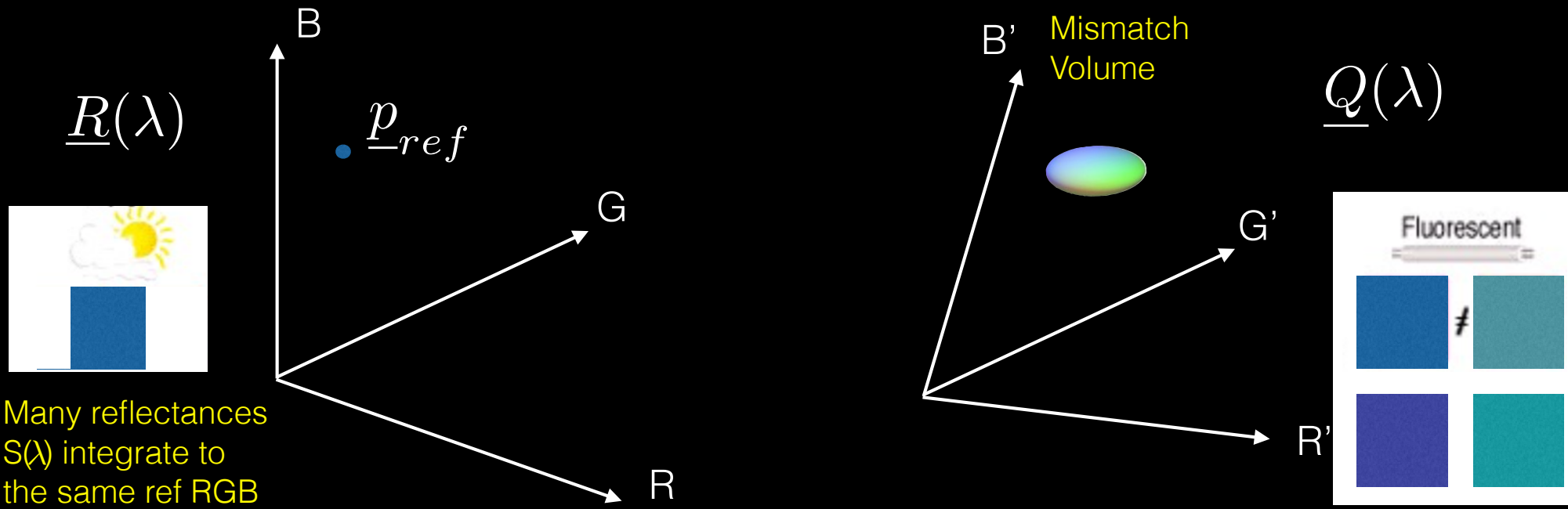


Solve for the 'Mismatch volume' analogous to how we solved for the OCS

$$\max_{S(\lambda)} \|\underline{n} \cdot \underline{q}\| \quad s.t. \quad \left\{ \right.$$

Metamer Mismatch Volume

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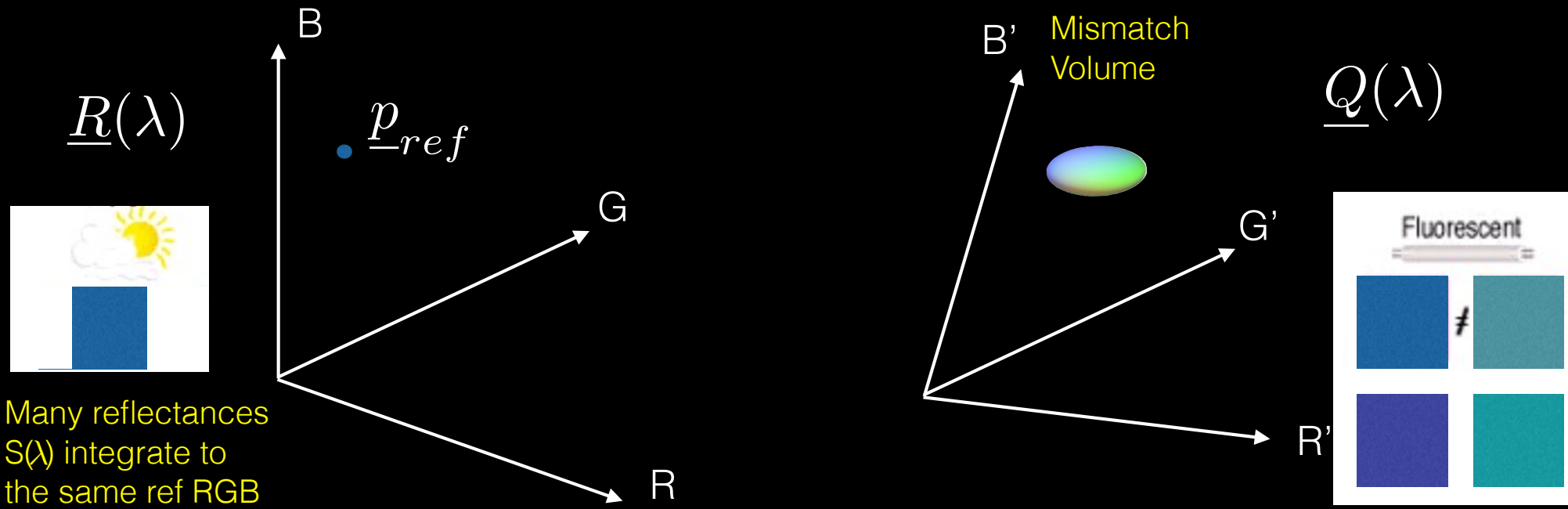


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Metamer Mismatch Volume

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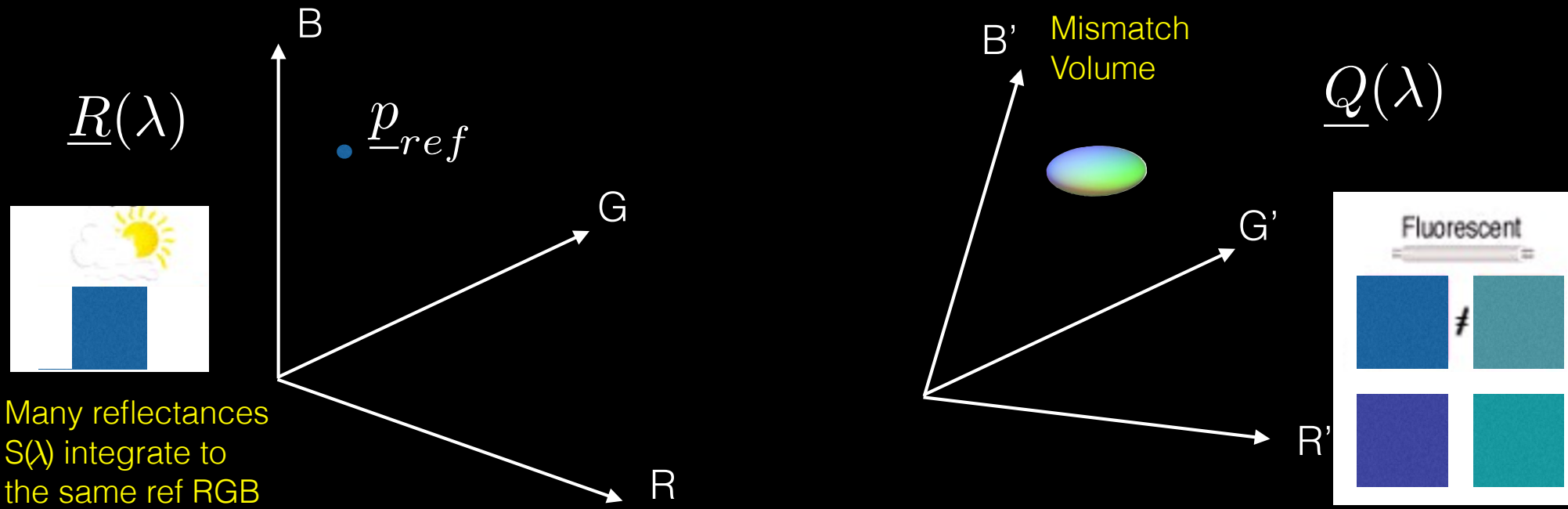
Many reflectances $S(\lambda)$ integrate to the same ref RGB

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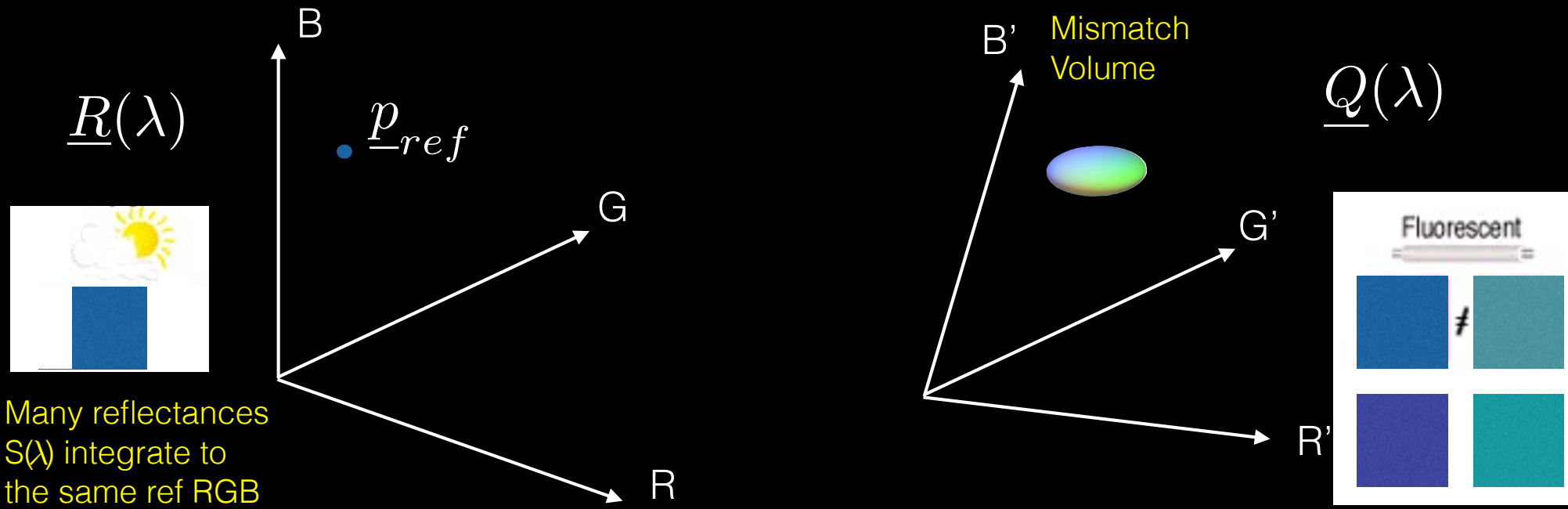
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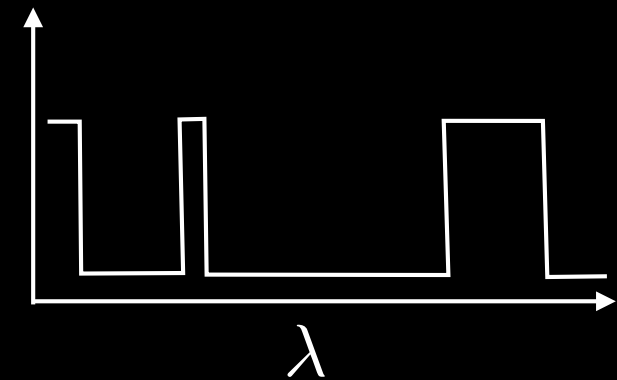
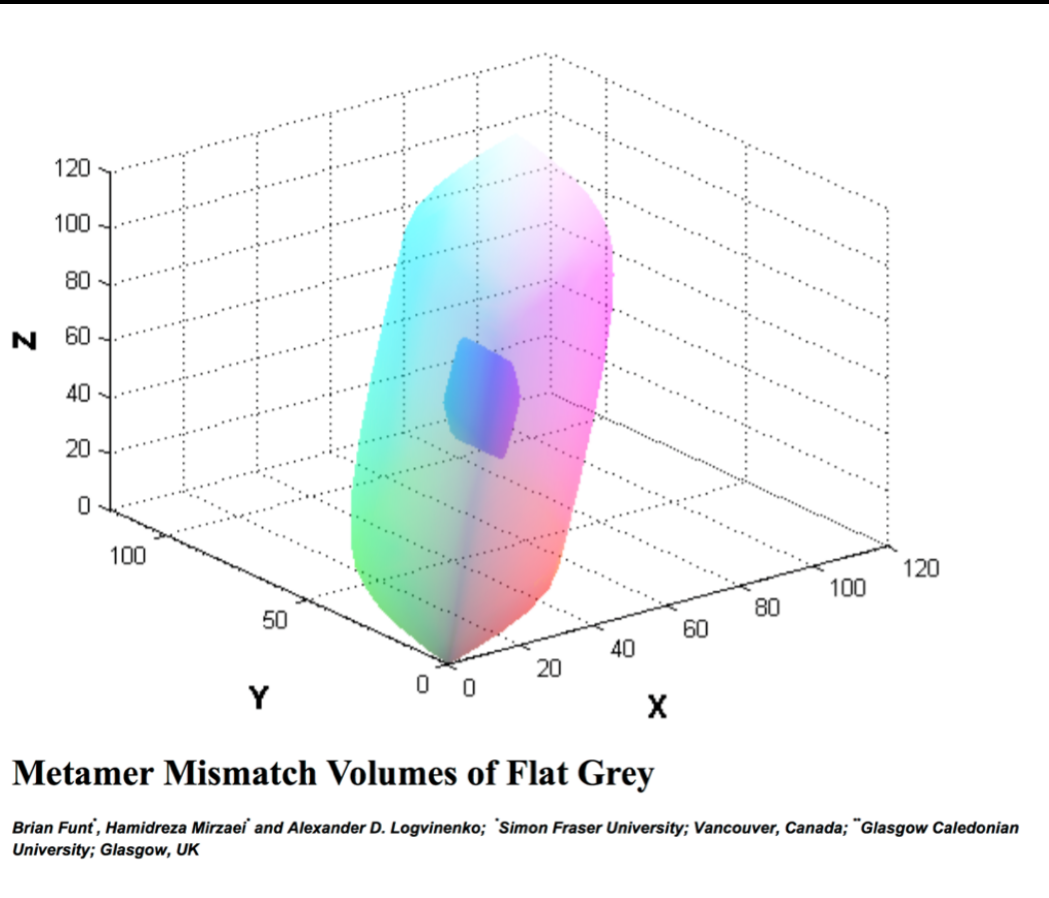
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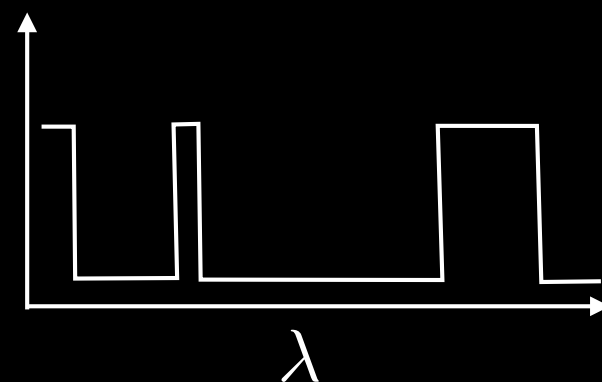
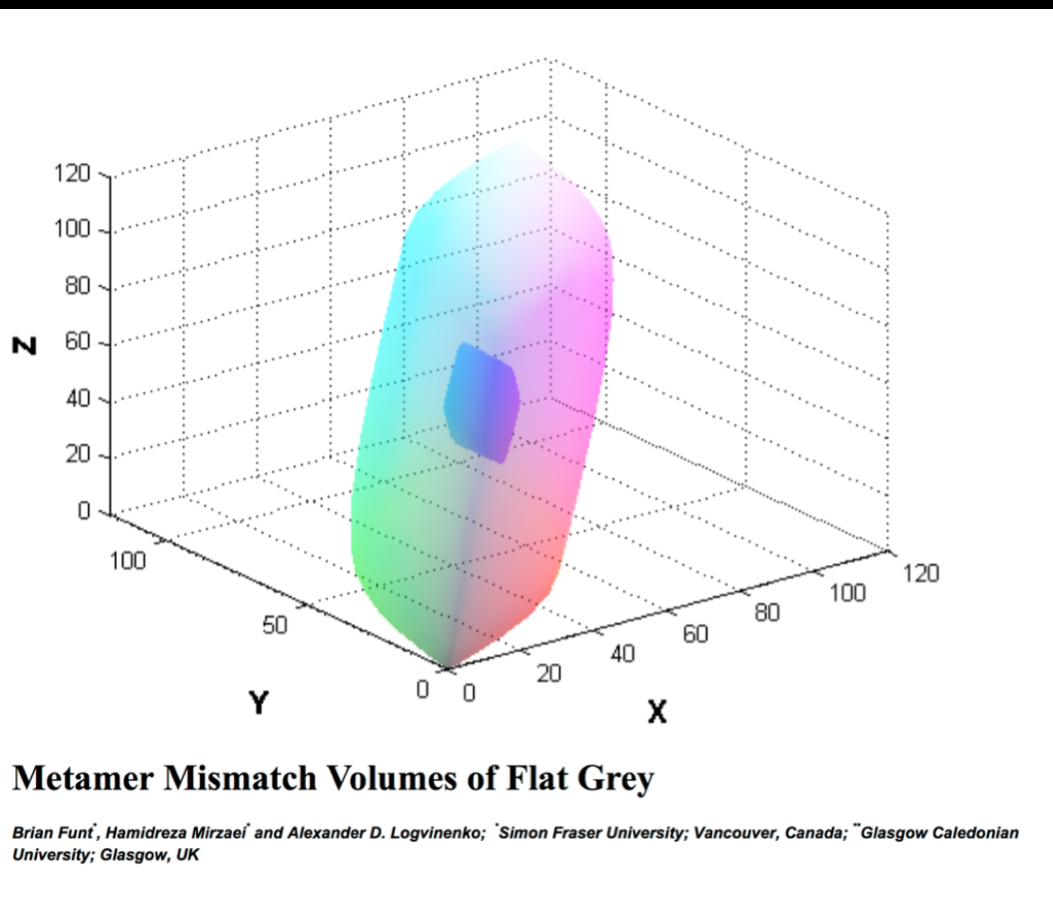
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Metamer Mismatch Volume



A lot of theory 'assumes' the reflectances on the boundary of the metamer mismatch volume is '0-1' and has 5 transitions

Metamer Mismatch Volume



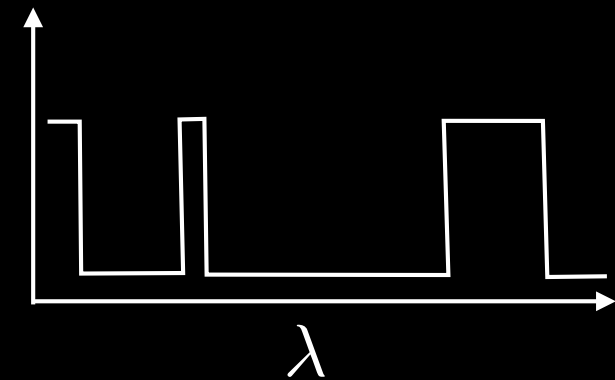
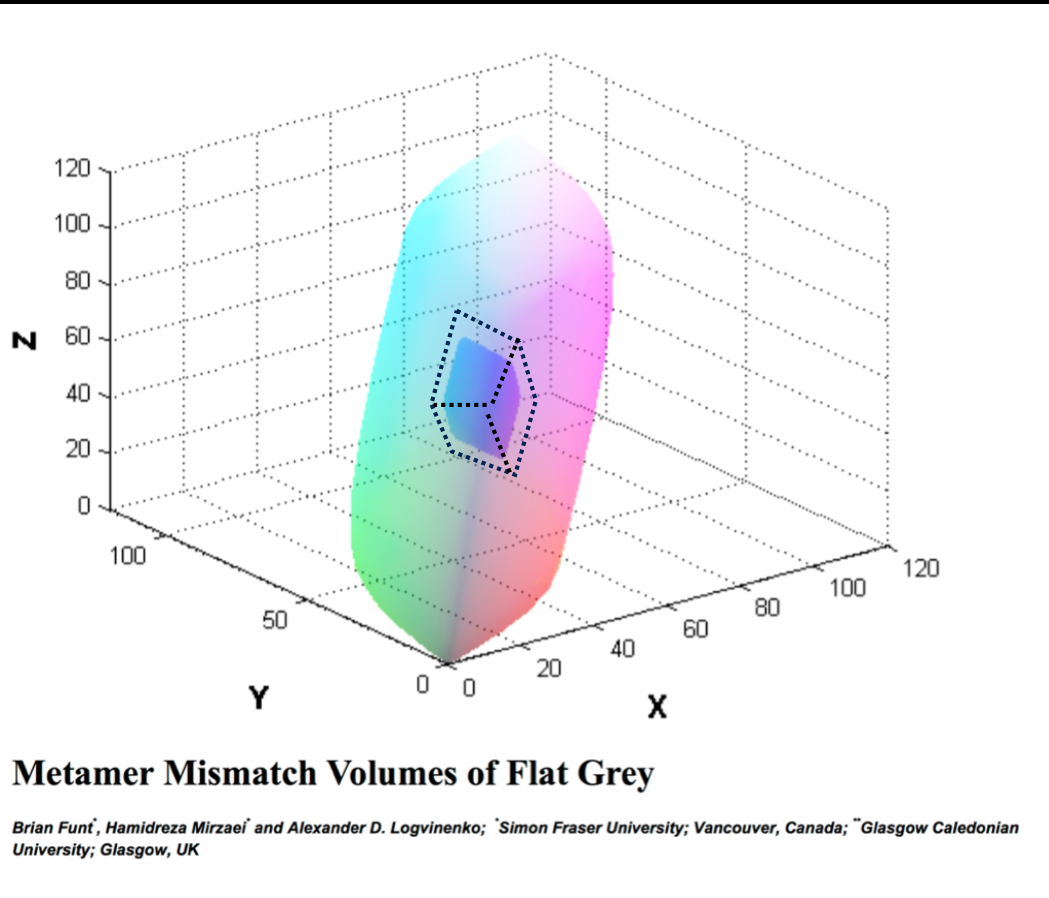
A lot of theory 'assumes' the reflectances on the boundary of the metamer mismatch volume is '0-1' and has 5 transitions

This assumption is wrong. Using the optimisation method for computing metamer sets produces 50% larger volumes.

Metamer mismatch volumes using spherical sampling

Michal Mackiewicz², Hans J. Rivertz², Graham D. Finlayson¹; ¹School of Computing Sciences, University of East Anglia, Norwich, UK; ²Norwegian University of Science and Technology, Trondheim, Norway

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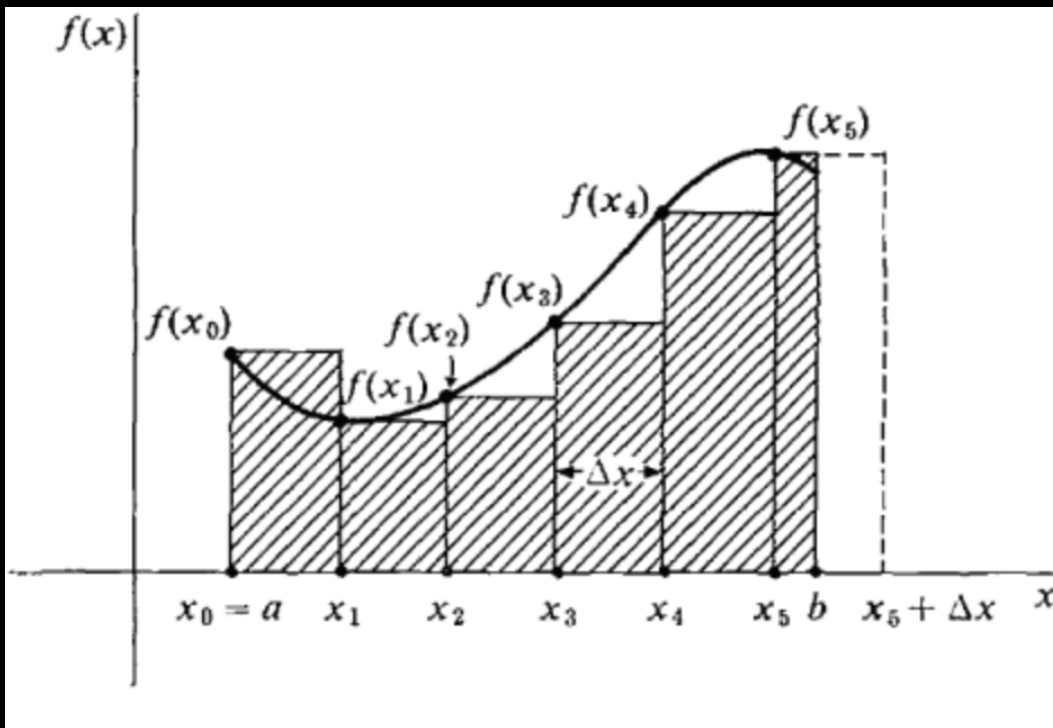
Metamer Sets:

Can we recover 'material' from images?



And does this help solve vision tasks?

Discretely ...



$$\int_{\omega} \underline{R}(\lambda) S(\lambda) d\lambda$$

$$\underline{\rho} = \mathcal{R}^t \underline{S}$$

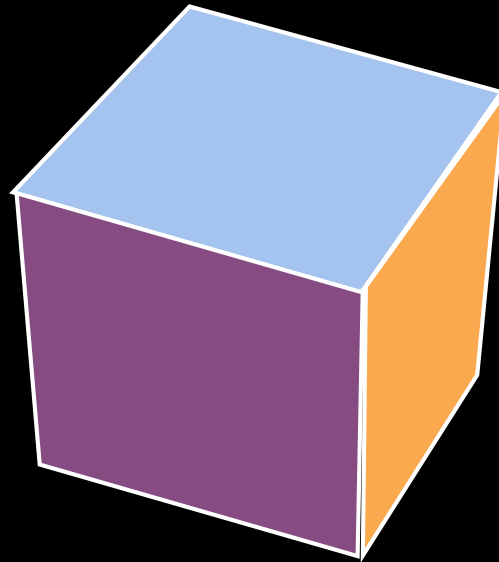
3x1 RGB

Nx3 matrix of 'effective' sensitivities

Nx1 reflectance vector

The Set of all Reflectances

Hyper-cube
constraint
(if 31 sample
wavelengths then
31 dimensional hypercube)



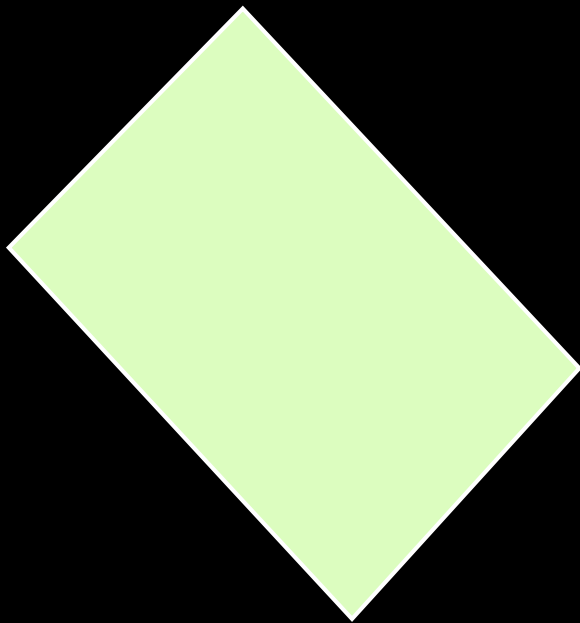
Reflectances that 'project' to \underline{p}

Reflectance hyper-plane

constraint: All vectors \underline{S}
that 'project' to the same
RGB lie on a plane

(if 31 sample
wavelengths then

28 dimensional affine hyperplane or 'flat')



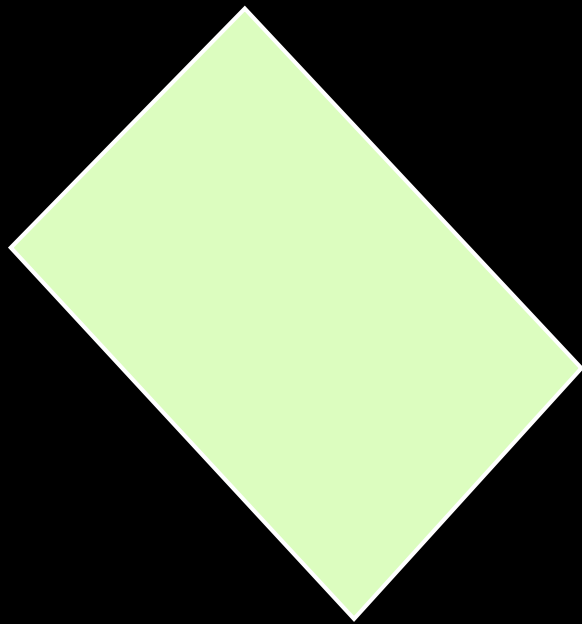
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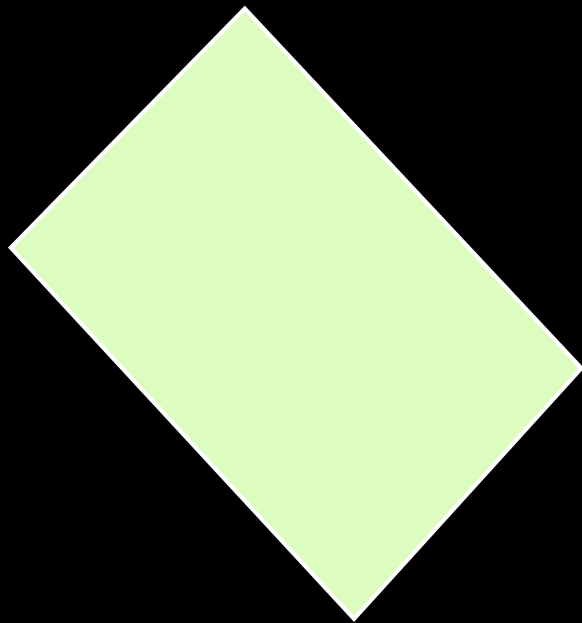
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$$\mathcal{R}^t \underline{S} = \underline{p} \equiv$$
$$\mathcal{R}^t [\mathcal{R} \underline{\alpha} + \mathcal{R}^\perp \underline{\beta}] = \underline{p}$$

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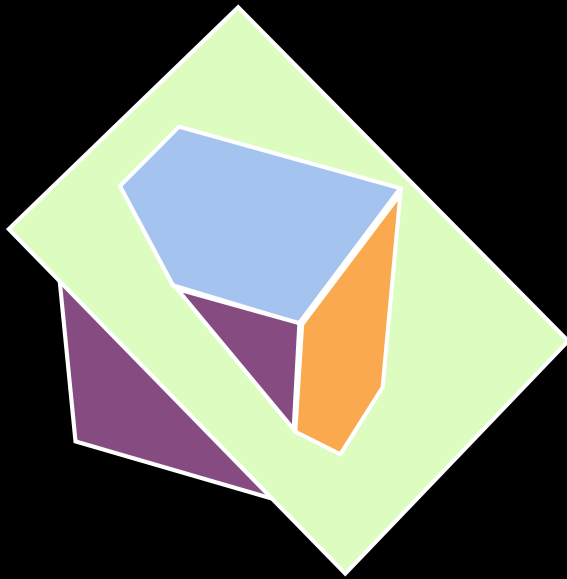
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$$\mathcal{R}^t \underline{S} = \underline{p} \equiv$$

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$$\underline{S}_{ref} + \sum_{i=1}^{28} Black_i(\lambda) \beta_i$$

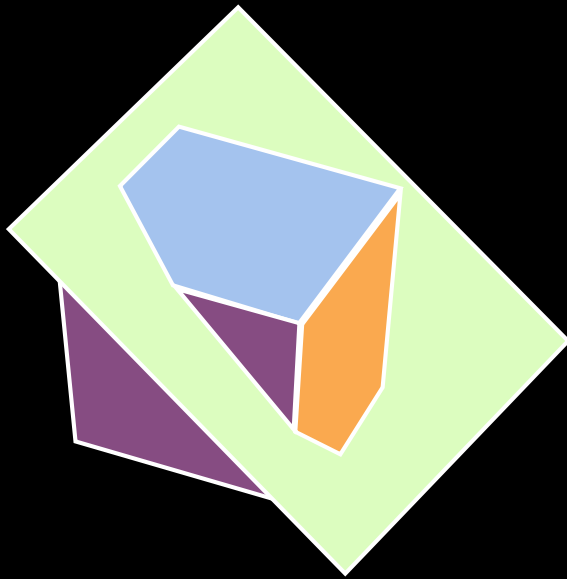
Metamer Sets



The Metamer Set is the intersection of a hyper-cube with a hyper-plane (can have complex geometry [thousands of points, computationally hard])

$$O(n^{d/2})$$

Metamer Sets

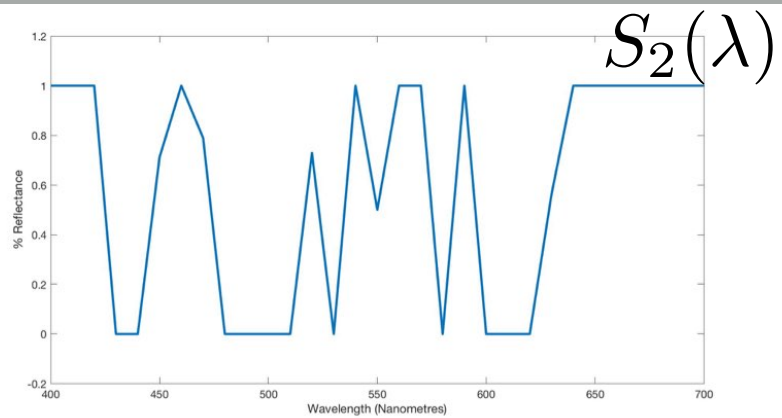
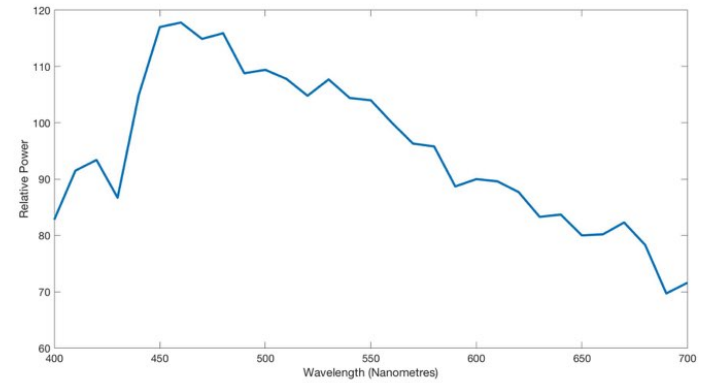
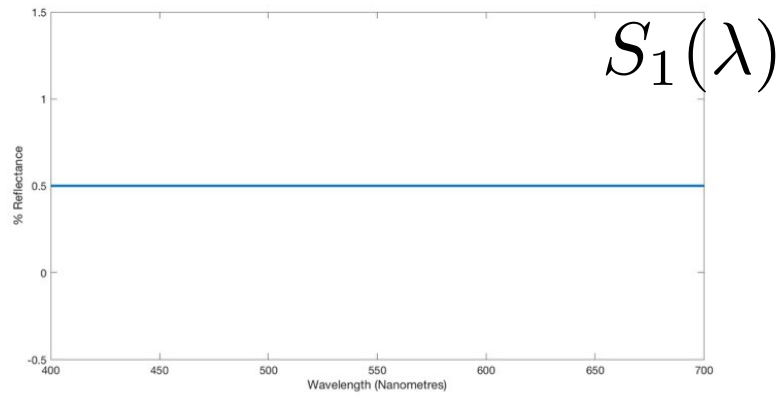


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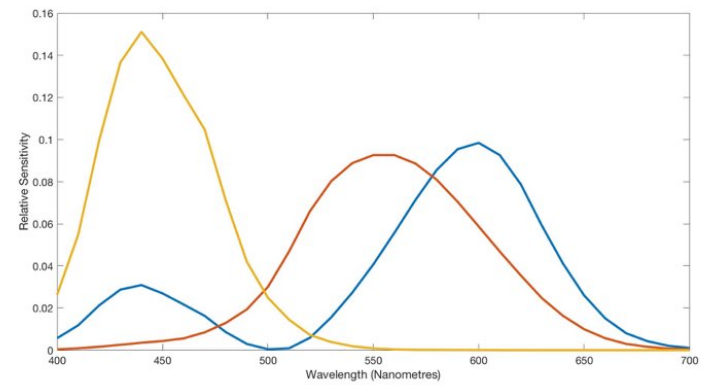
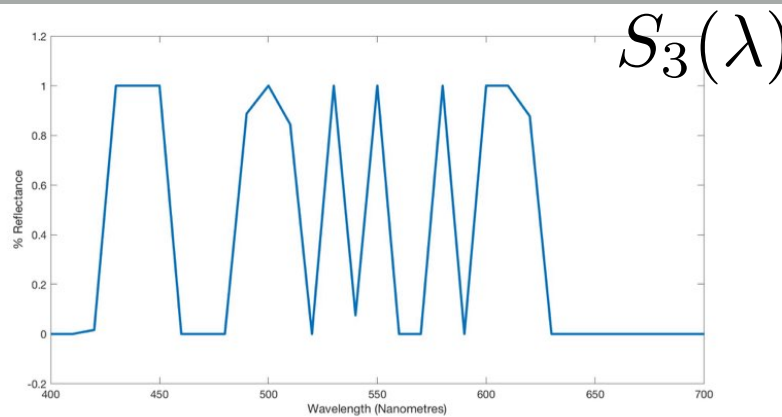
$$O(n^{d/2})$$

A hypercube is the intersection of half spaces delimited by the cube's faces (hyperplanes)

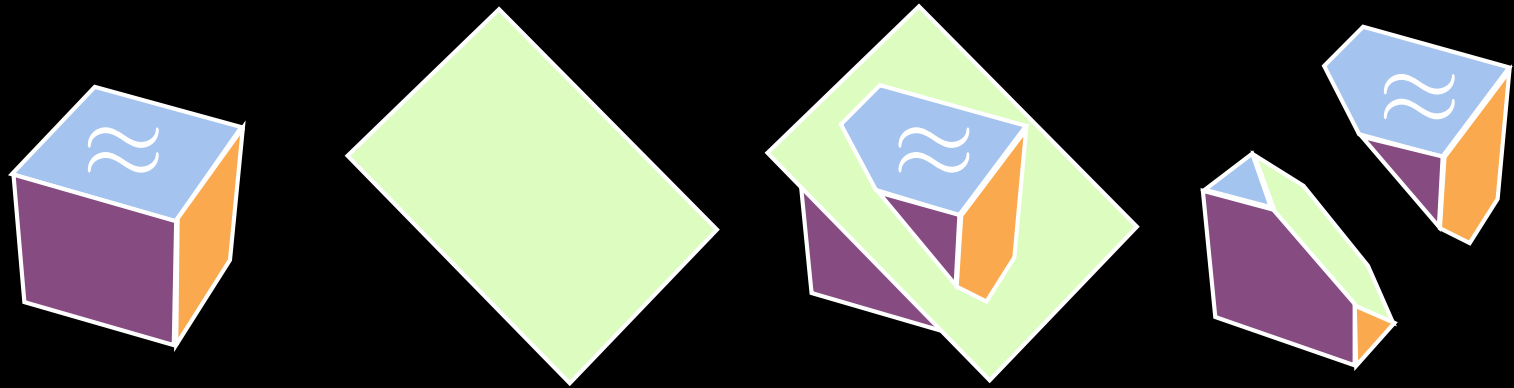
Intersection of half spaces in N-D is found using ND convex hull algorithm



$$\int_{\omega} \underline{Q}(\lambda) E(\lambda) S(\lambda) d\lambda = [47 \ 49 \ 53]$$



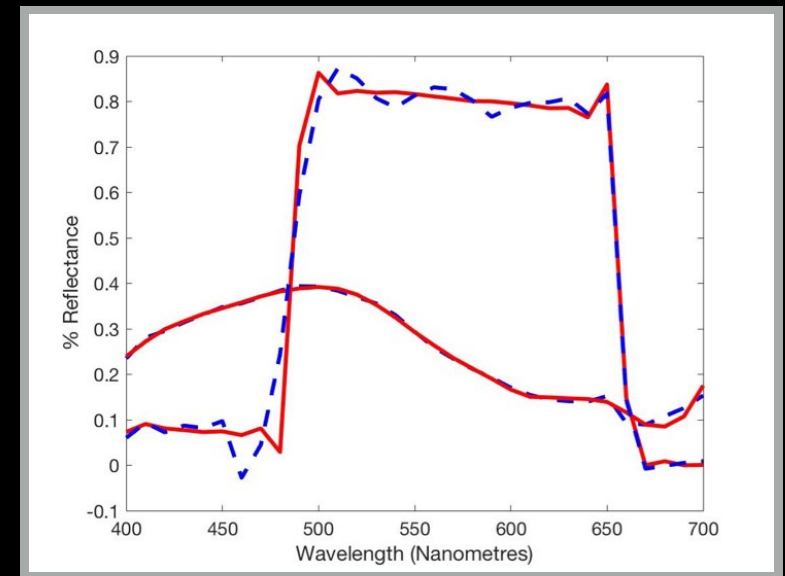
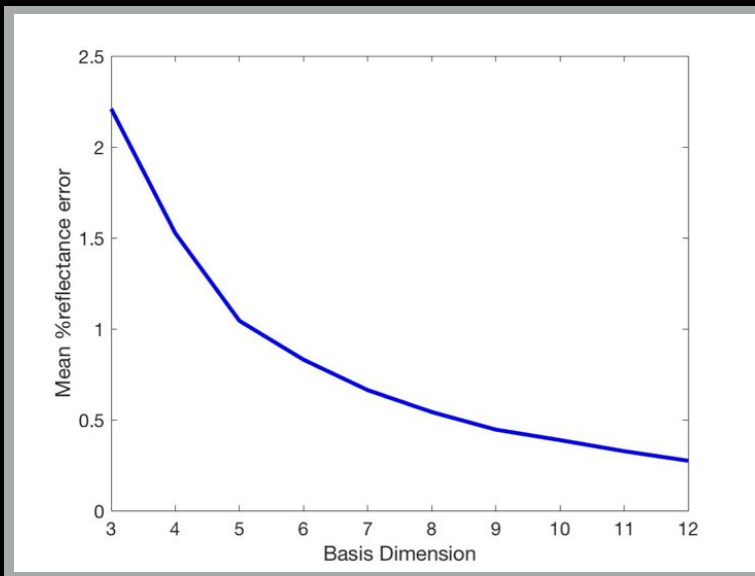
Basic Metamer Sets



denotes N-dimensional reflectances that 'live' inside the 31-d hypercube ($N \ll 31$)

Basis Approximations

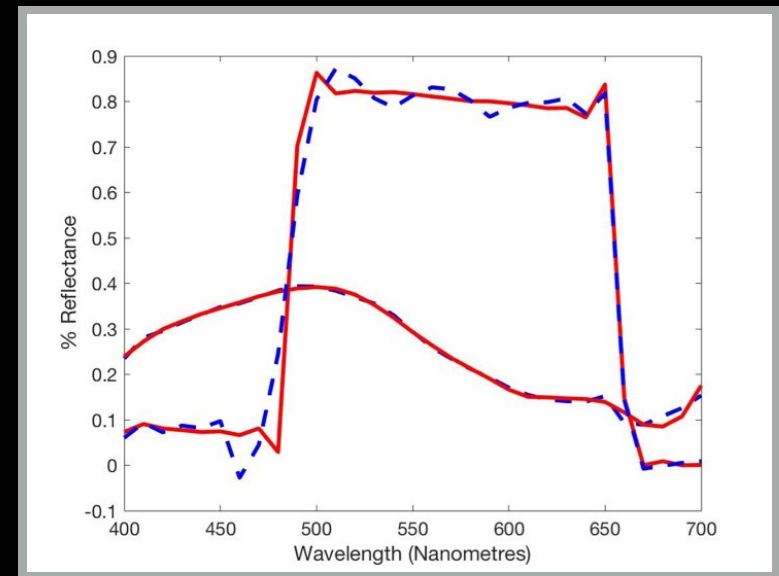
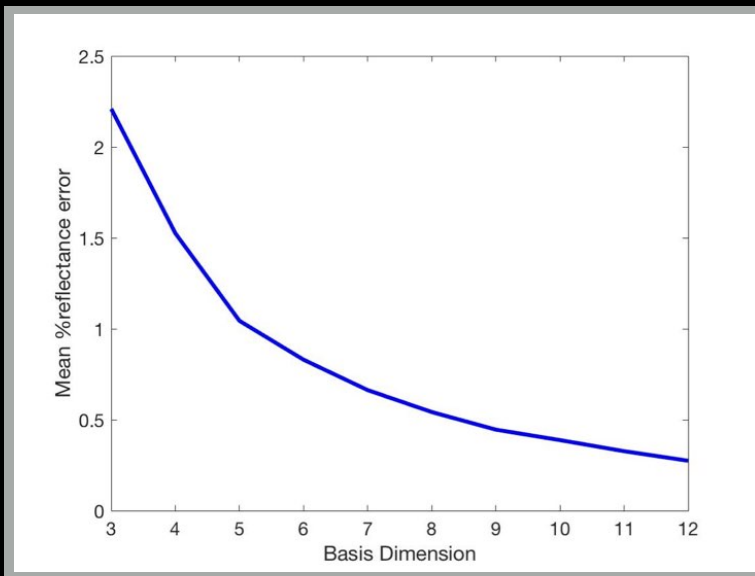
$$S(\lambda) \approx \sum_{i=1}^N \sigma_i S_i(\lambda)$$



Basis Approximations

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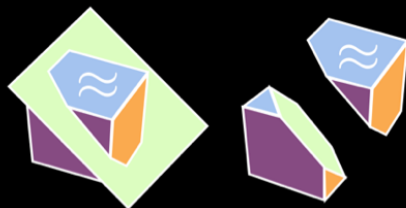
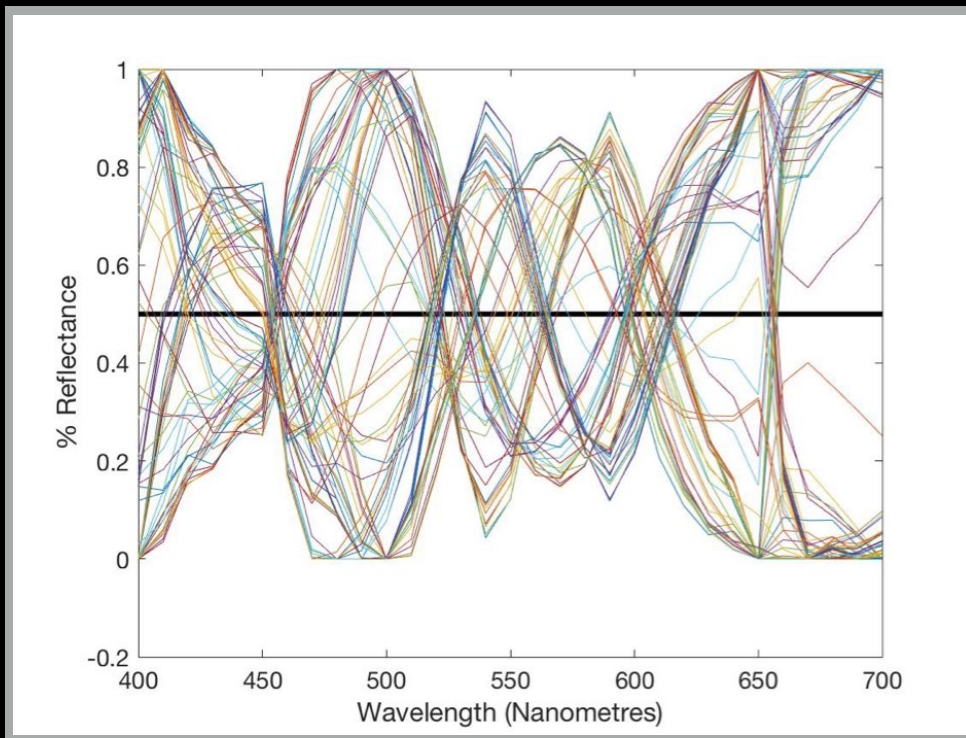
How many basis functions are needed?



Shown: 'extremal' metamers for grey reflectance assuming 7 dimensional CVA basis set

These are solved for *exactly* because

i) hypercube constraints in 7D are defined by 14 hyperplanes

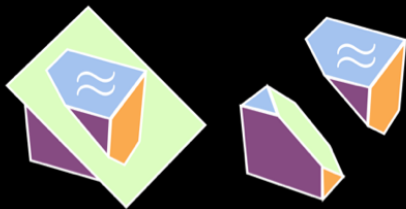
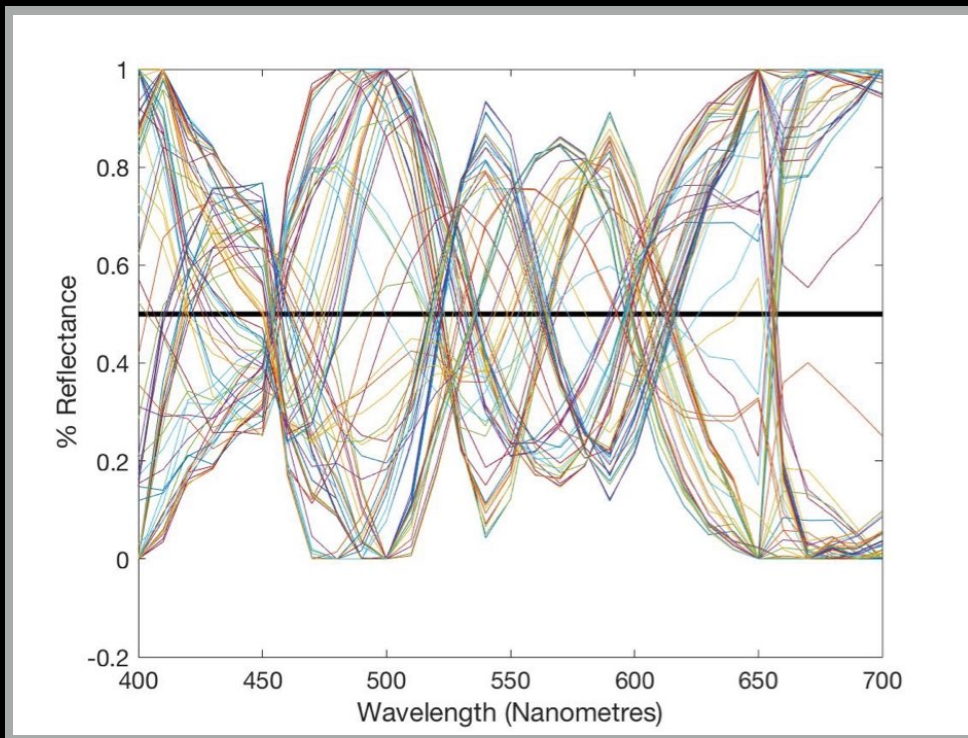


Shown: 'extremal' metamers for grey reflectance assuming 7 dimensional CVA basis set

These are solved for *exactly* because

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ii) Intersecting the 14 cube hyperplanes with the **reflectance hyperplane** can be computed using high dimensional convex hull



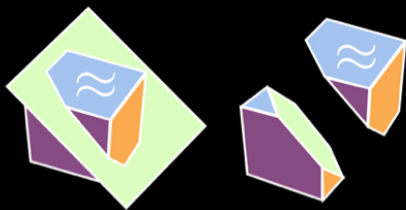
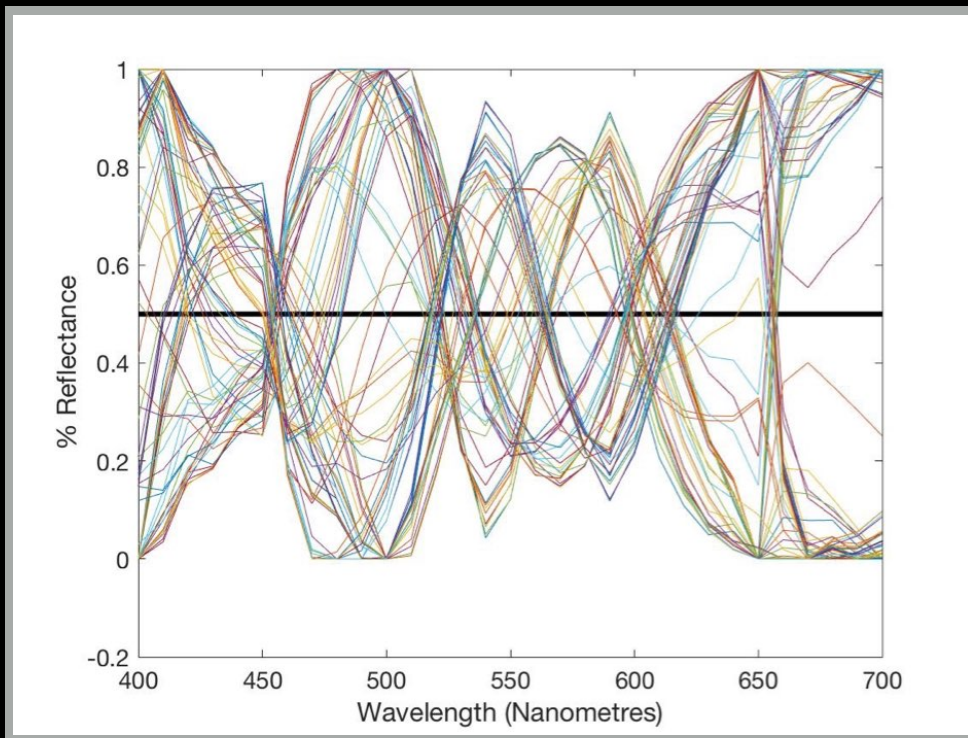
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iii) Complexity $\sim O(\text{floor}([D-3]/2))$



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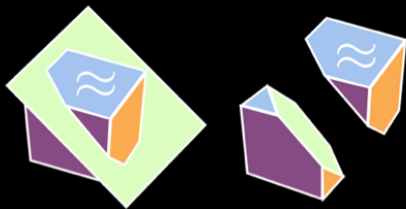
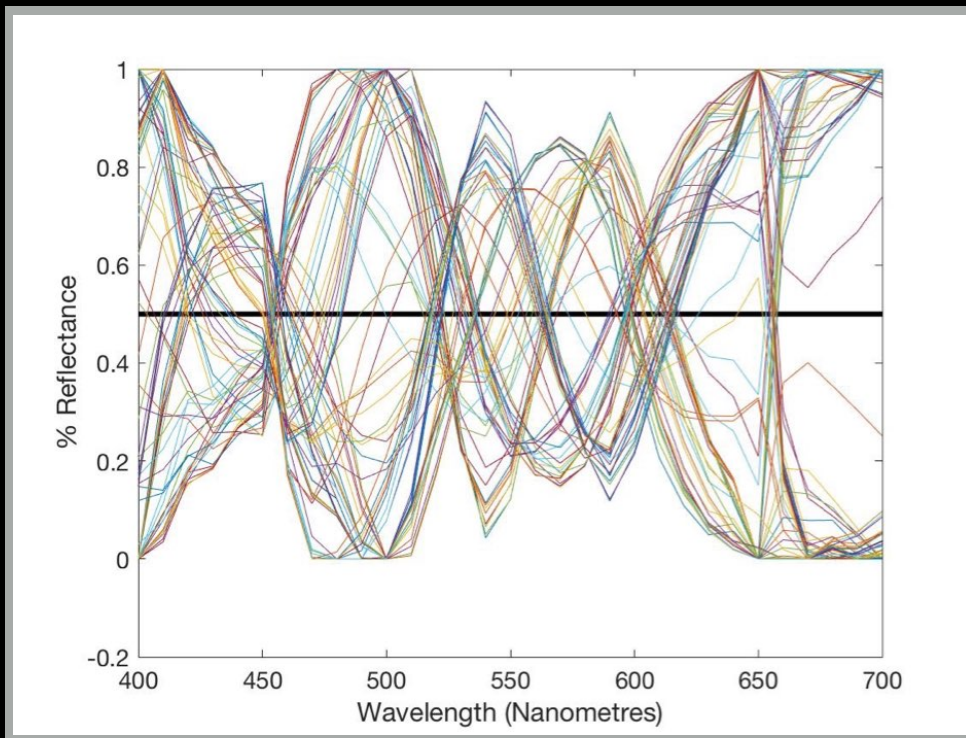
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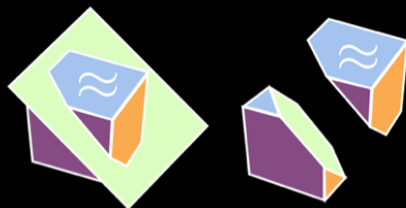
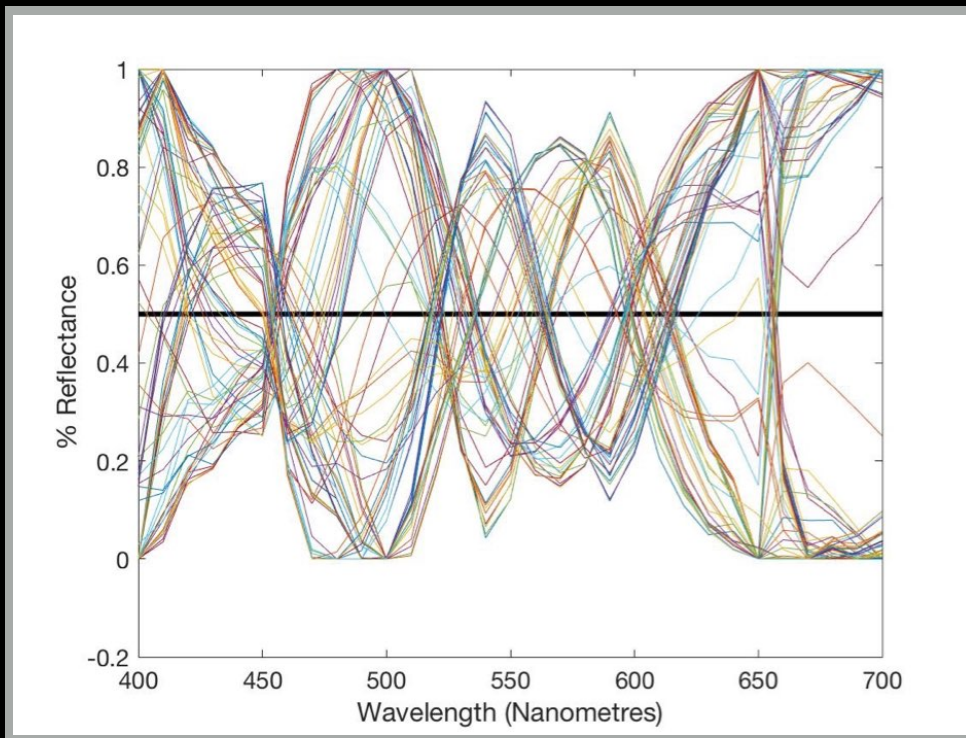
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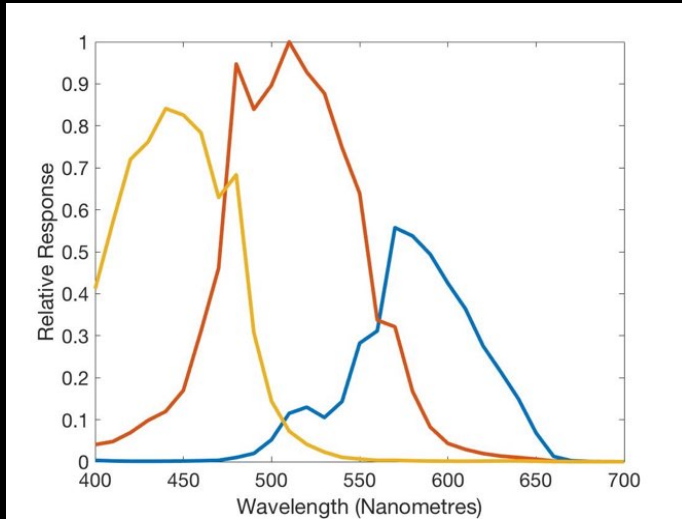
iv) 7D = $O(n^2)$, 12D = $O(n^3)$

v) Can calculate basic metamer sets for 12,13,14 dimensional basis (more degrees of freedom than typically used)



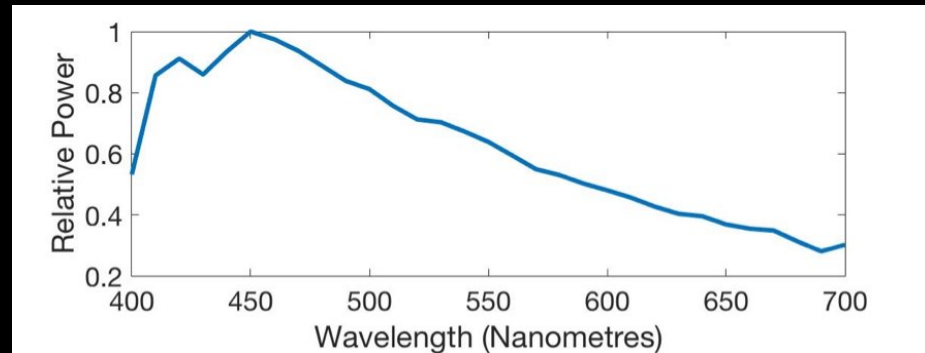
Worked Example

$$Q(\lambda)$$

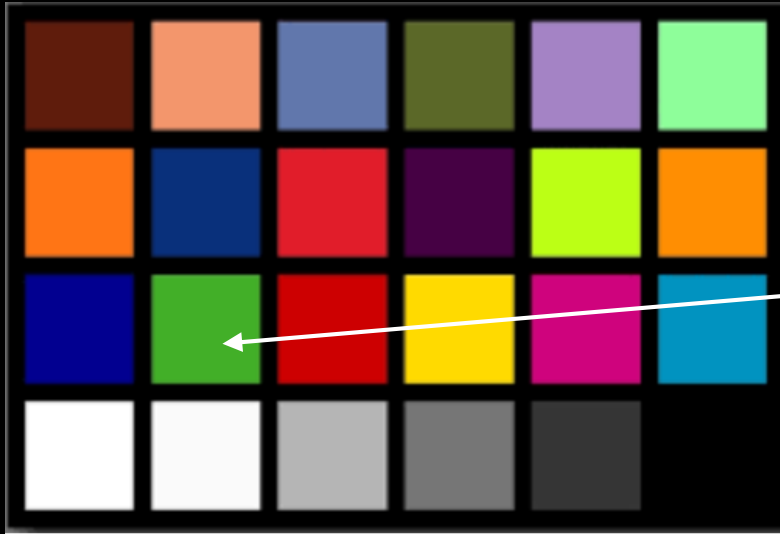
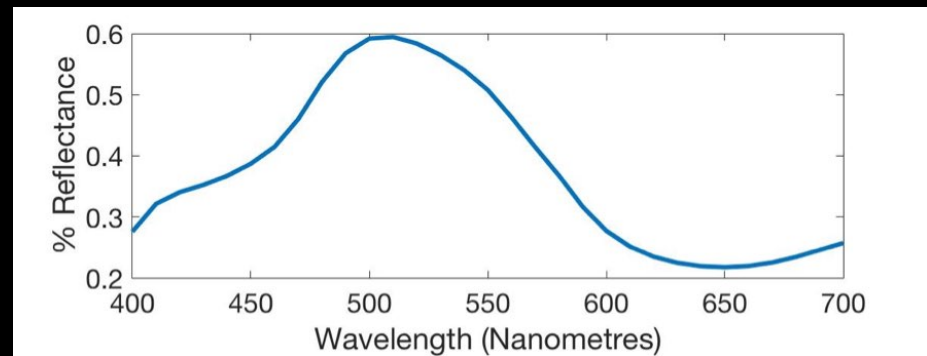


Canon D1
Spectral Sensitivities

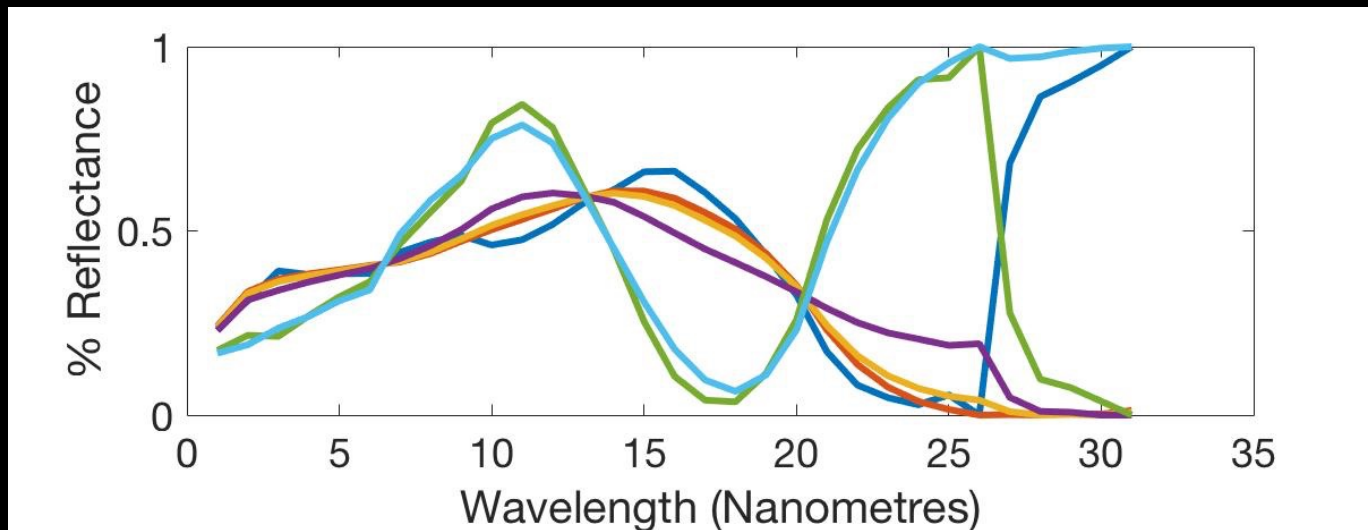
$$E(\lambda)$$



$$S(\lambda)$$



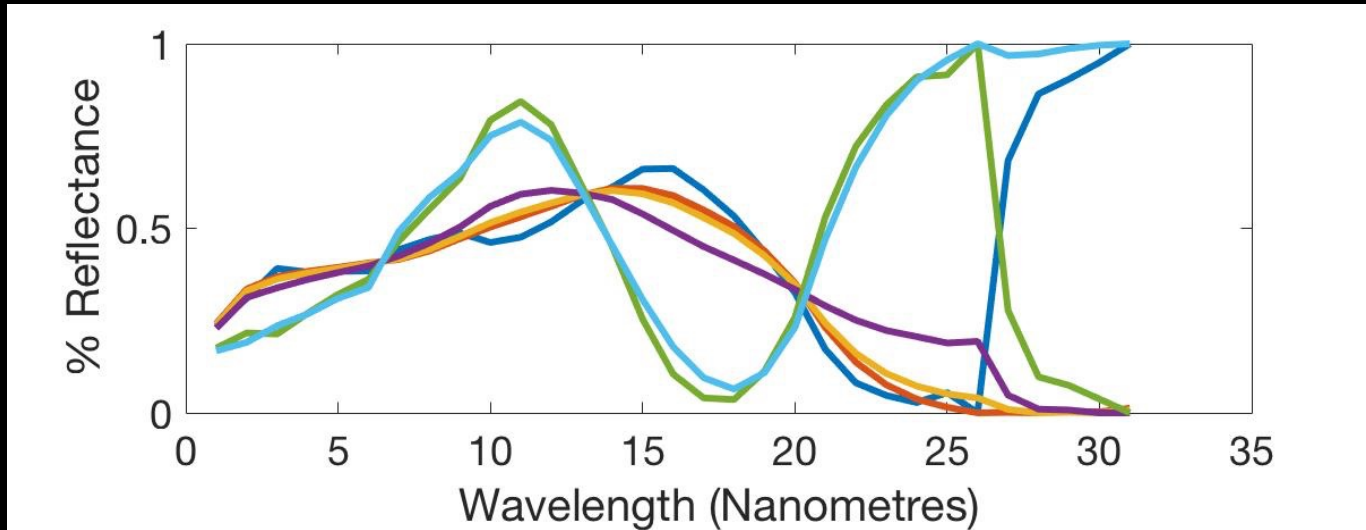
$$\int \underline{Q}(\lambda) E(\lambda) S(\lambda) d\lambda = [1.19 \quad 4.24 \quad 2.56]$$



The spectra shown lie on the convex hull of the 'Metamer Set'

All spectra integrate to [1.19 4.24 2.45]

All convex combinations integrate to [1.19 4.24 2.45]

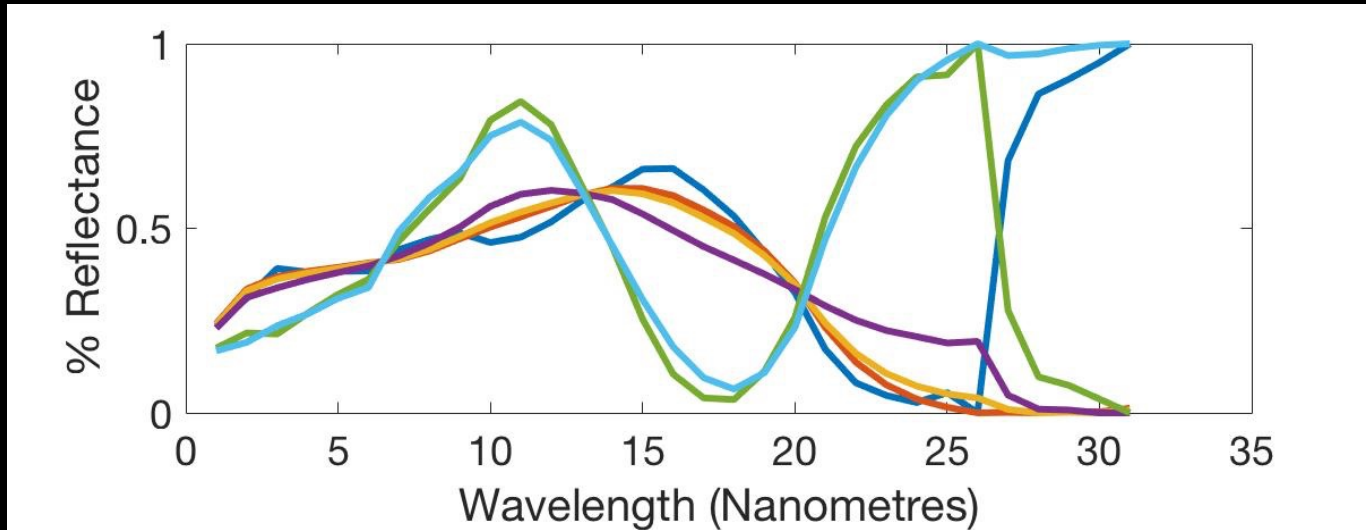


We are trying to characterise all reflectances which integrate to a single RGB.

$$\mathcal{R}^t \underline{S} = \underline{p} \equiv$$

$$\mathcal{R}^t [\mathcal{R}_\alpha \underline{\alpha} + \mathcal{R}^\perp \underline{\beta}] = \underline{p}$$

$$\underline{S}_{ref} + \sum_{i=1}^2 Black_i(\lambda) \beta_i$$



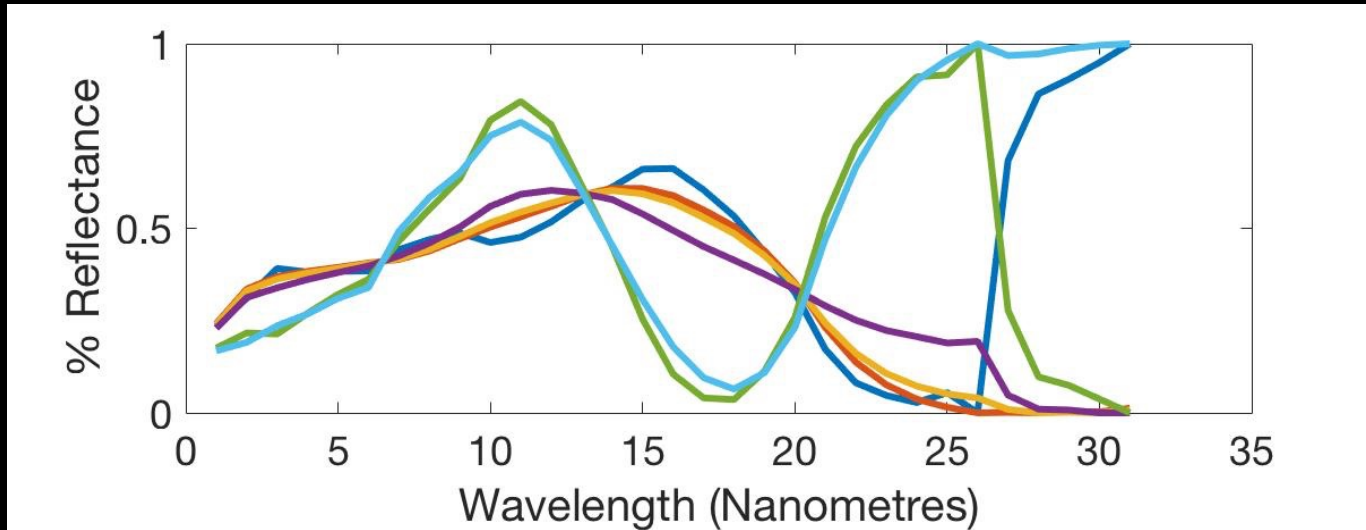
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The set comprises a ref spectrum that projects to the desired RGB and a combination of spectra that is orthogonal to the sensor space

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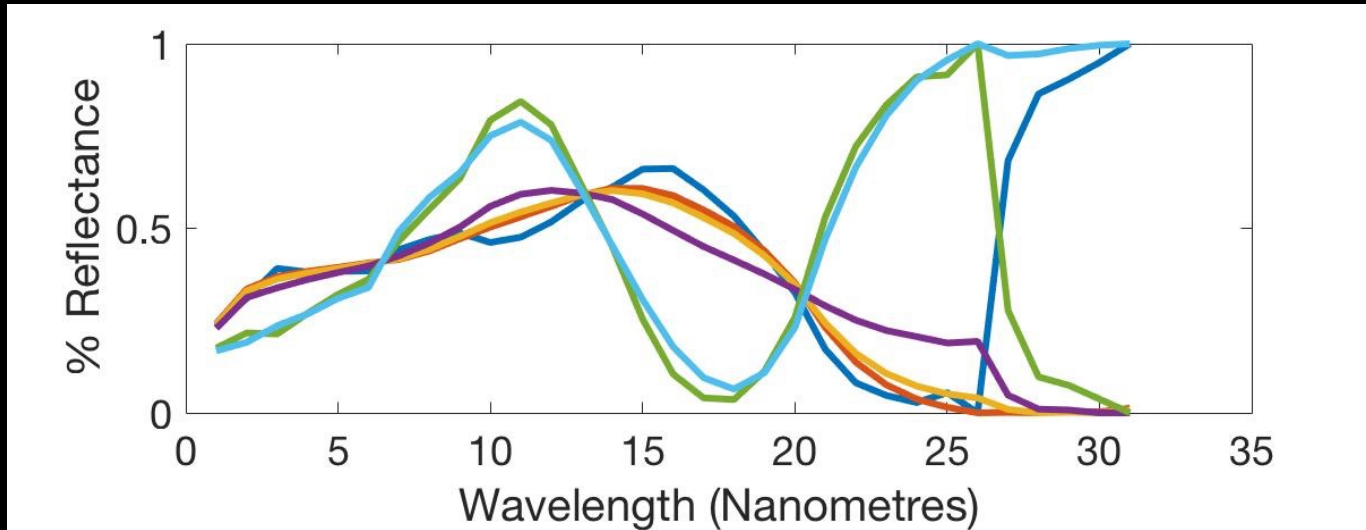
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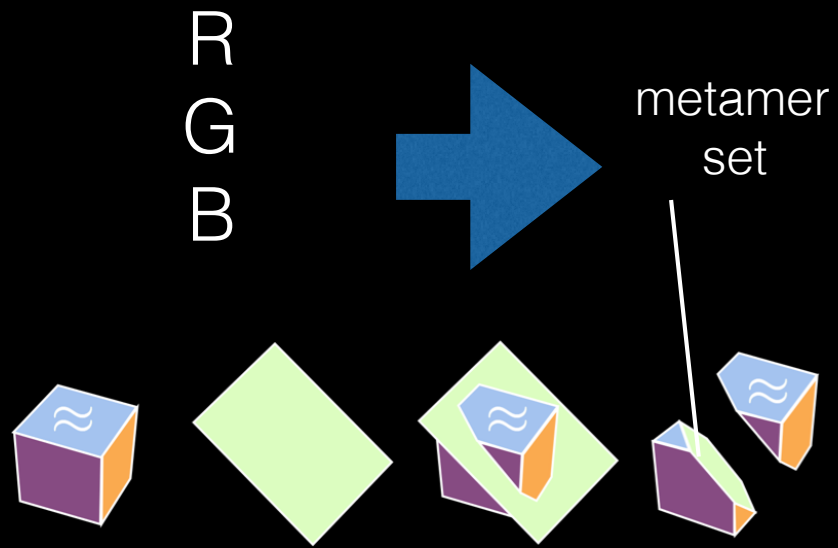
The metamer set is a 2-dimensional hyperplane in 5D (which in turn is embedded in 31-D)

$$\mathcal{R}^t \underline{S} = \underline{p} \equiv$$

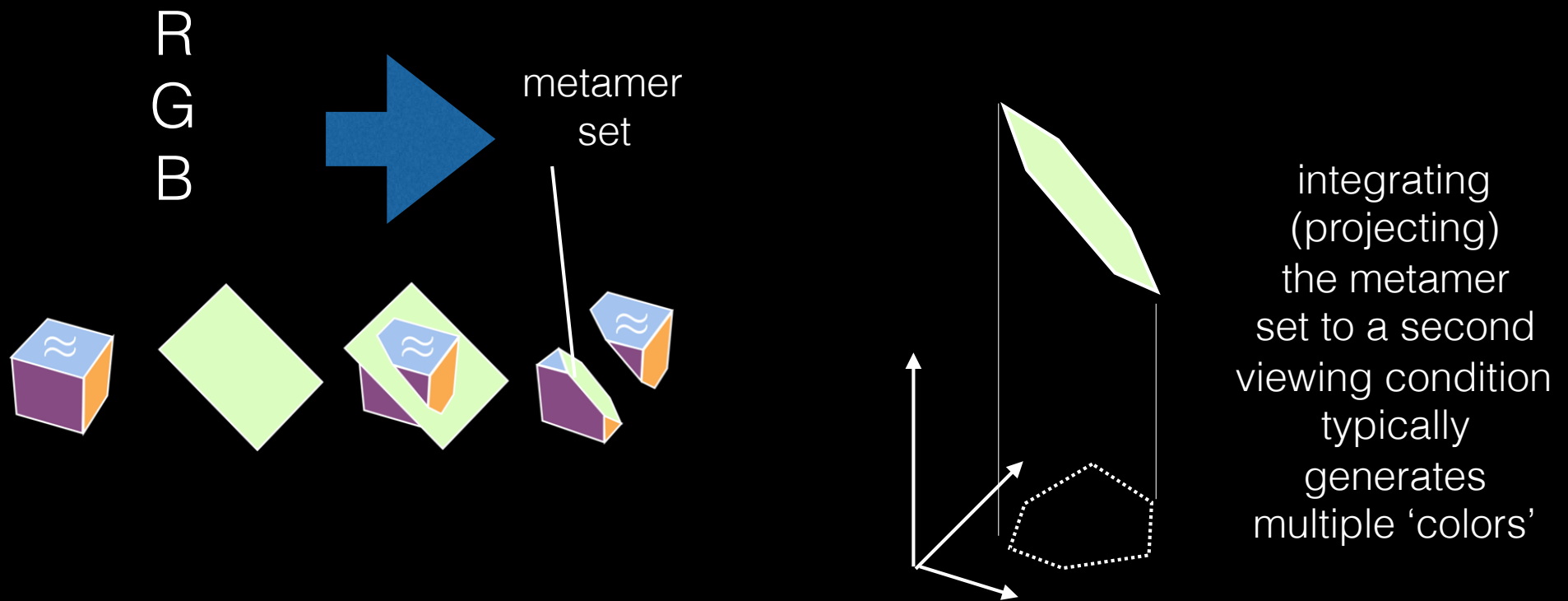
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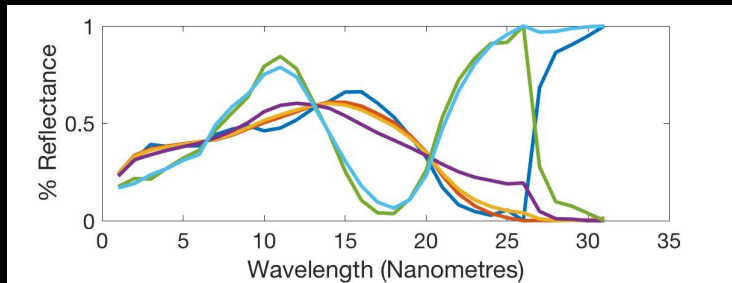
Reprojecting



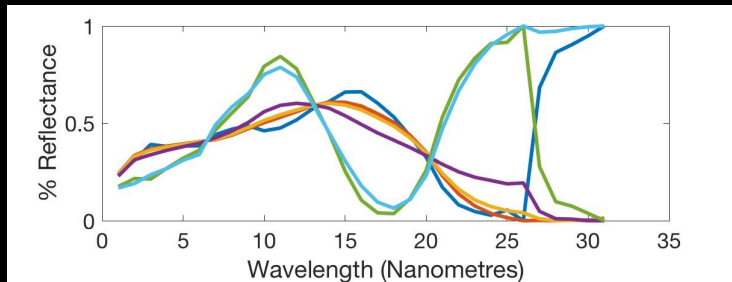
Reprojecting



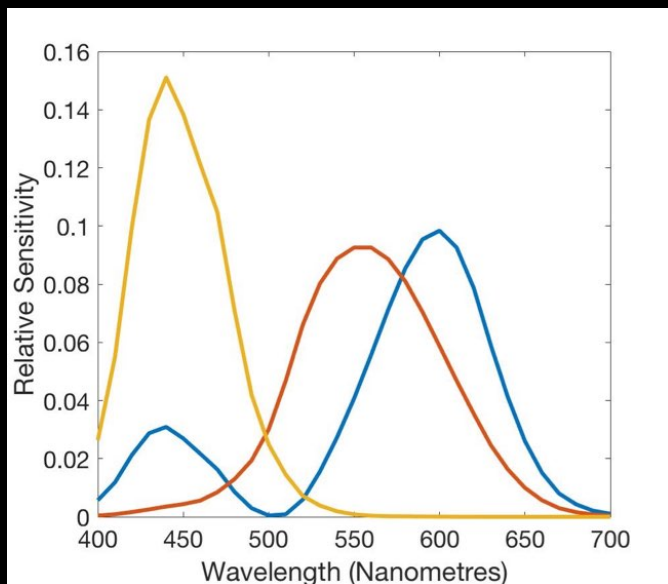
Re-projecting to XYZ



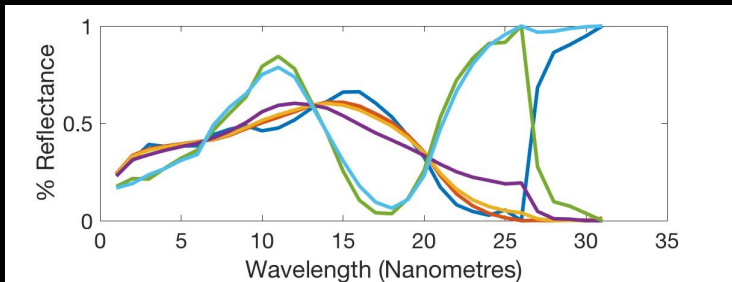
Re-projecting to XYZ



\int



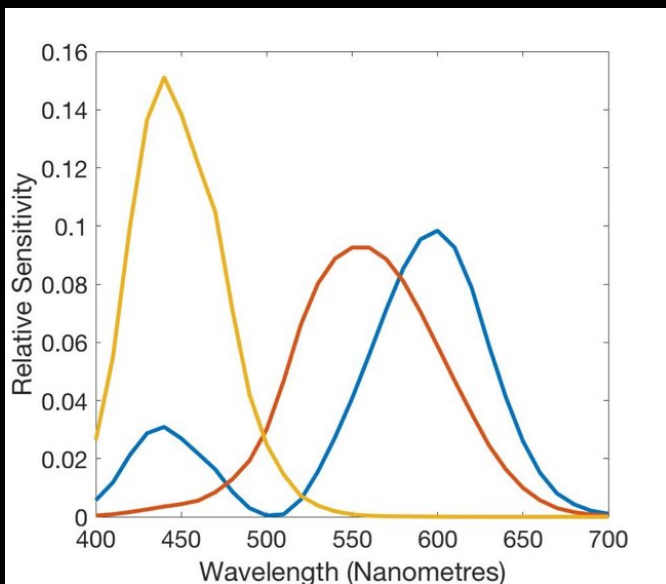
Re-projecting to XYZ



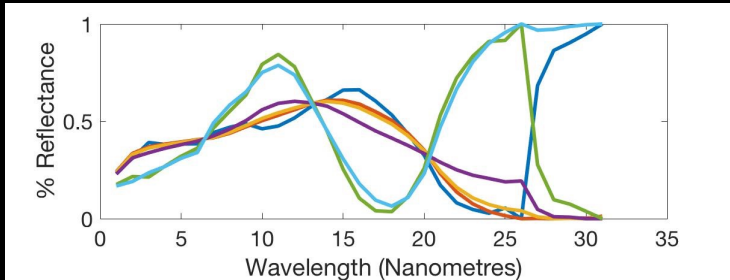
=

0.25	0.37	0.38
0.24	0.36	0.38
0.25	0.36	0.38
0.25	0.35	0.38
0.30	0.30	0.38
0.31	0.32	0.38

\int



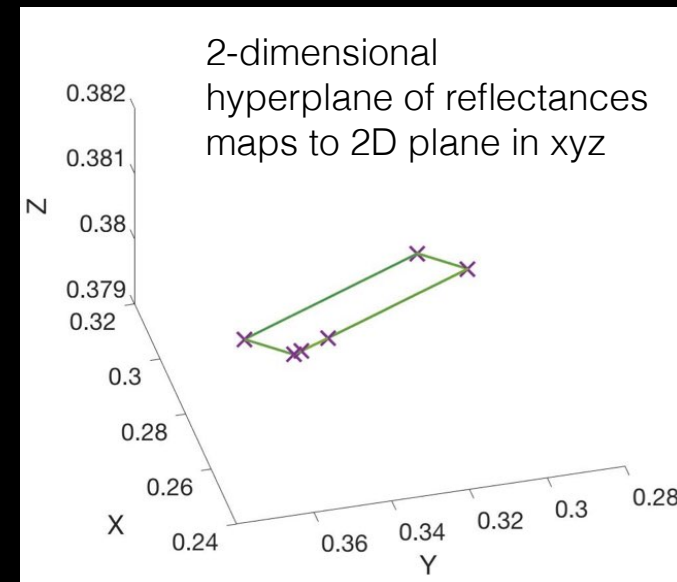
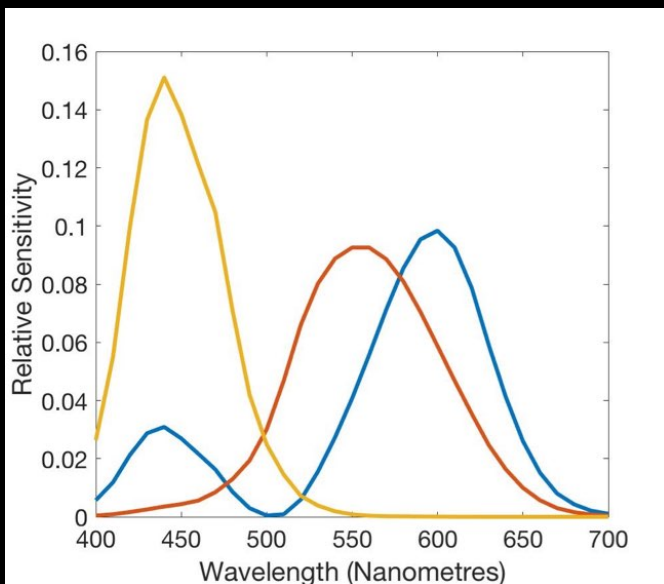
Re-projecting to XYZ

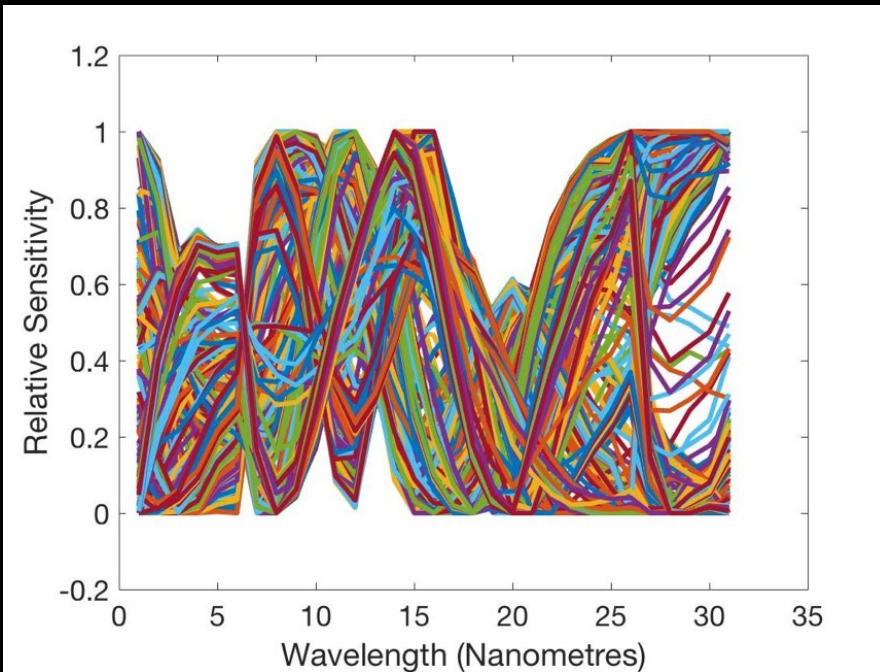


=

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\int

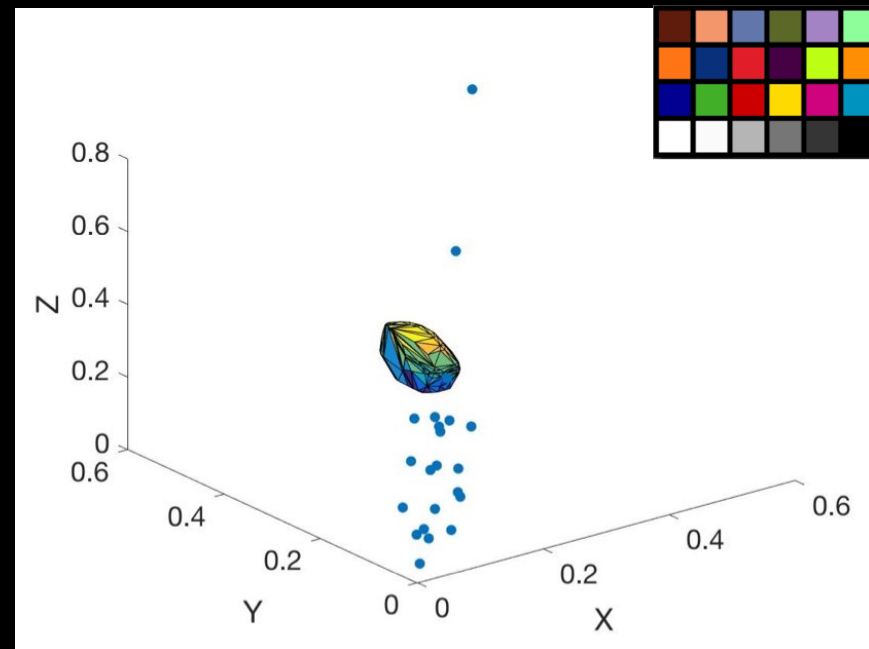
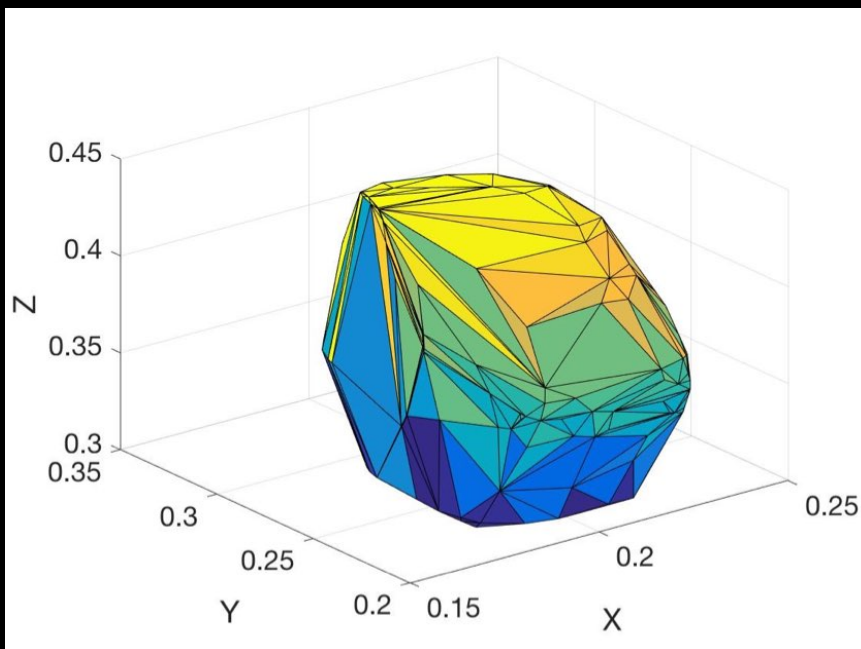




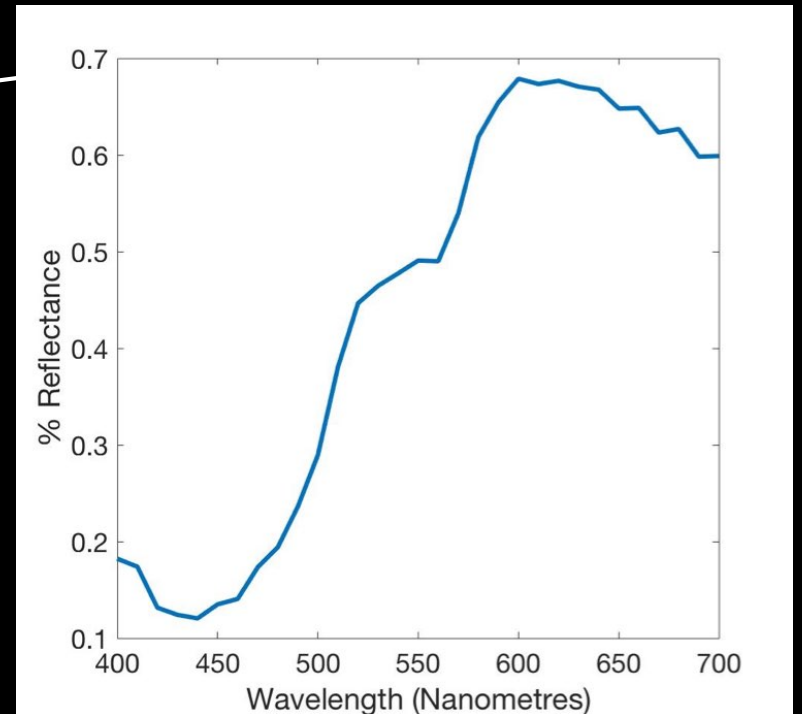
8 dimensional reflectance basis

yields 5 dimensional bounded convex region (in 8-d space) of metamers.

All of which integrate to [1.19 4.24 2.56]

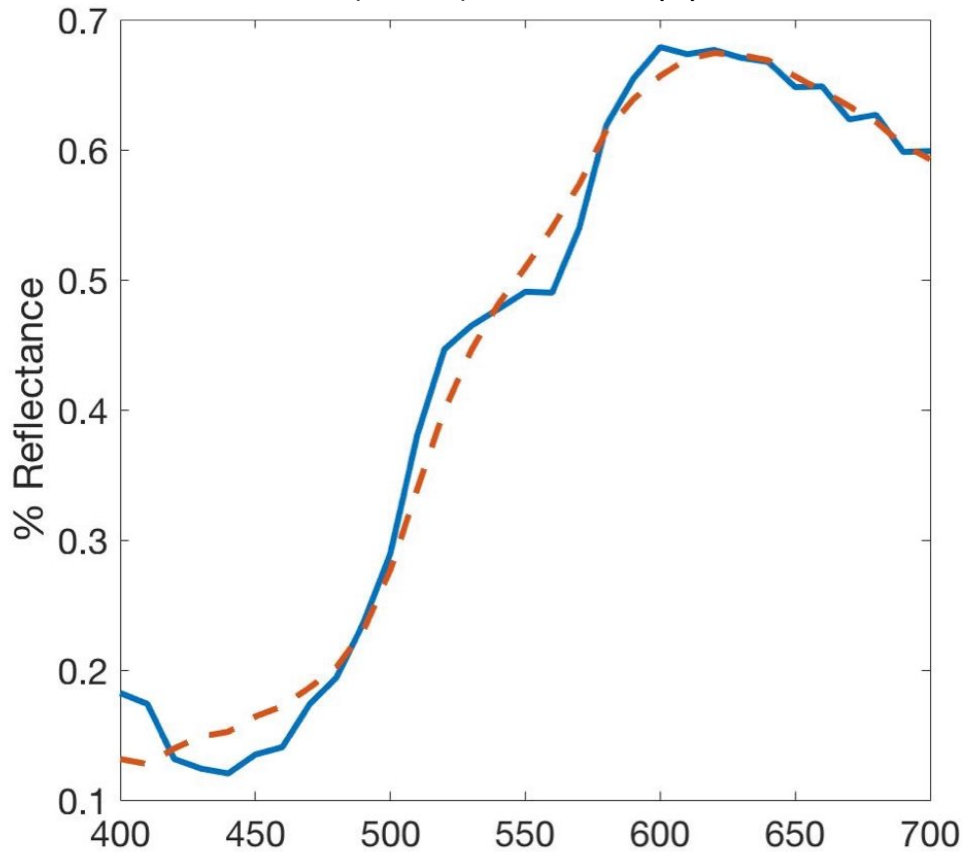


RGB to Spectra



How Many Dimensions?

Actual (blue) vs 3D approx

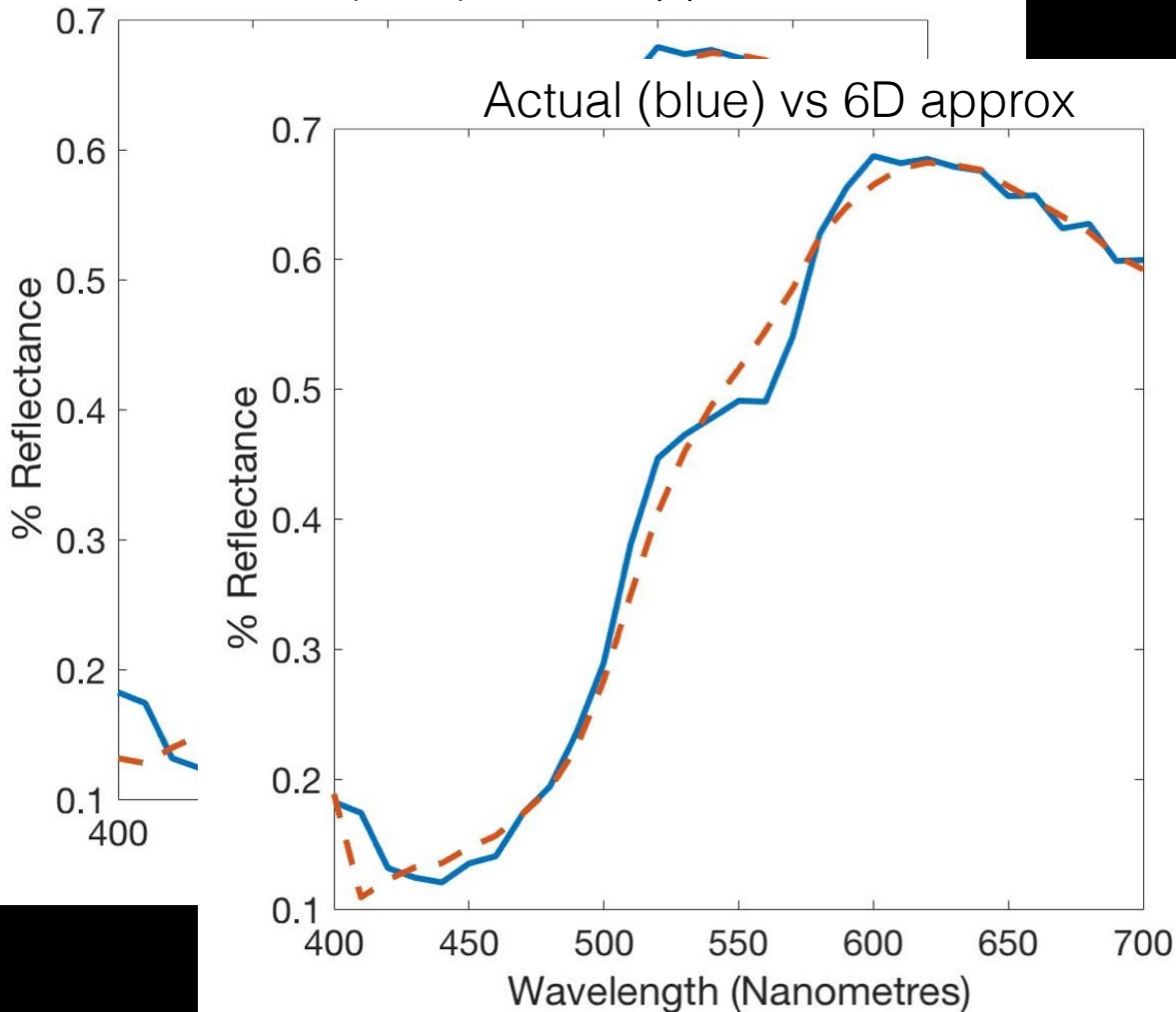


— ground-truth

— closest in X-D metamer set

How Many Dimensions?

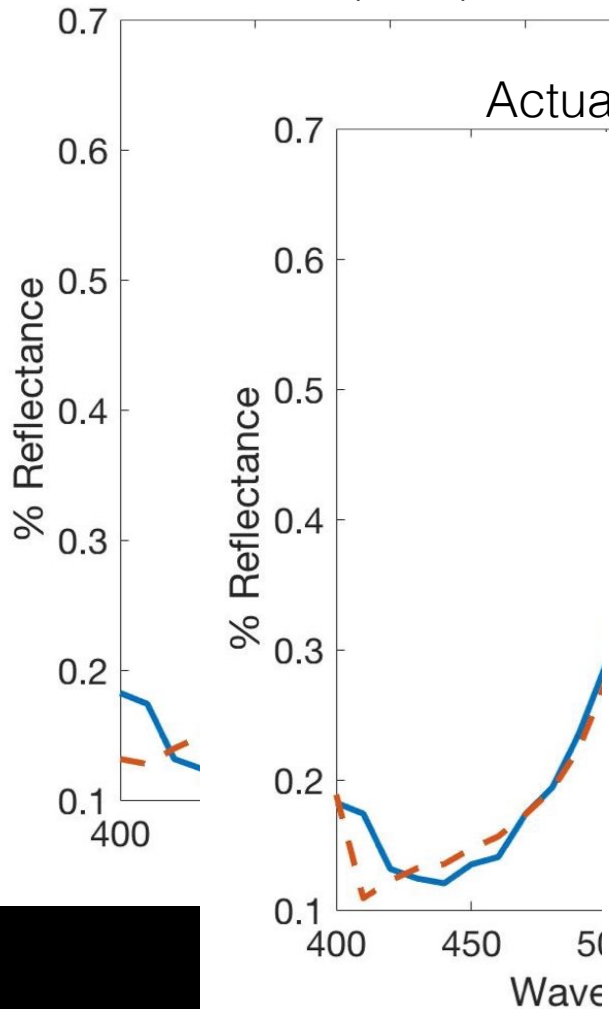
Actual (blue) vs 3D approx



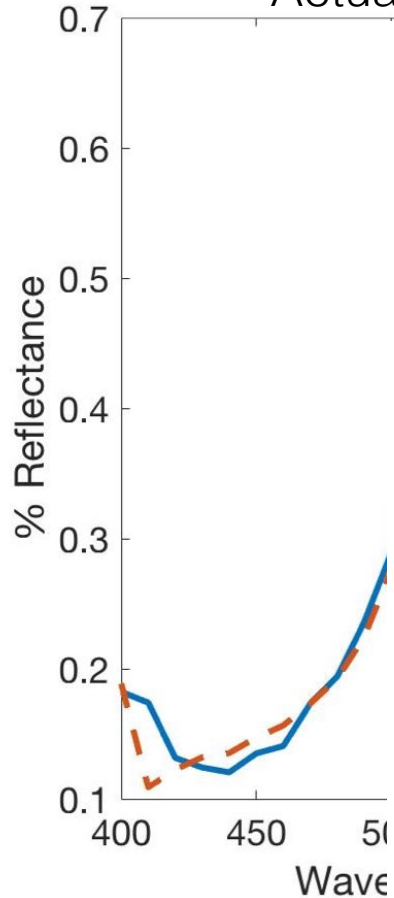
- ground-truth
- closest in X-D metamer set

How Many Dimensions?

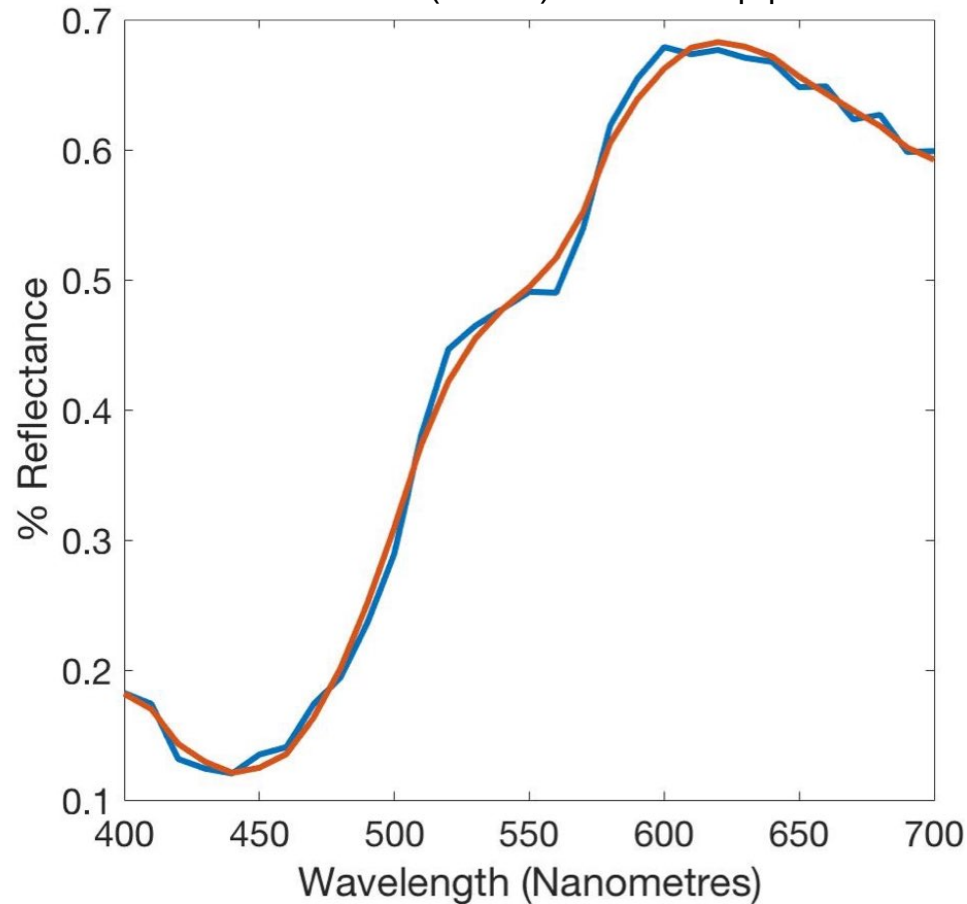
Actual (blue) vs 3D approx



Actual (blue) vs 6D approx



Actual (blue) vs 9D approx

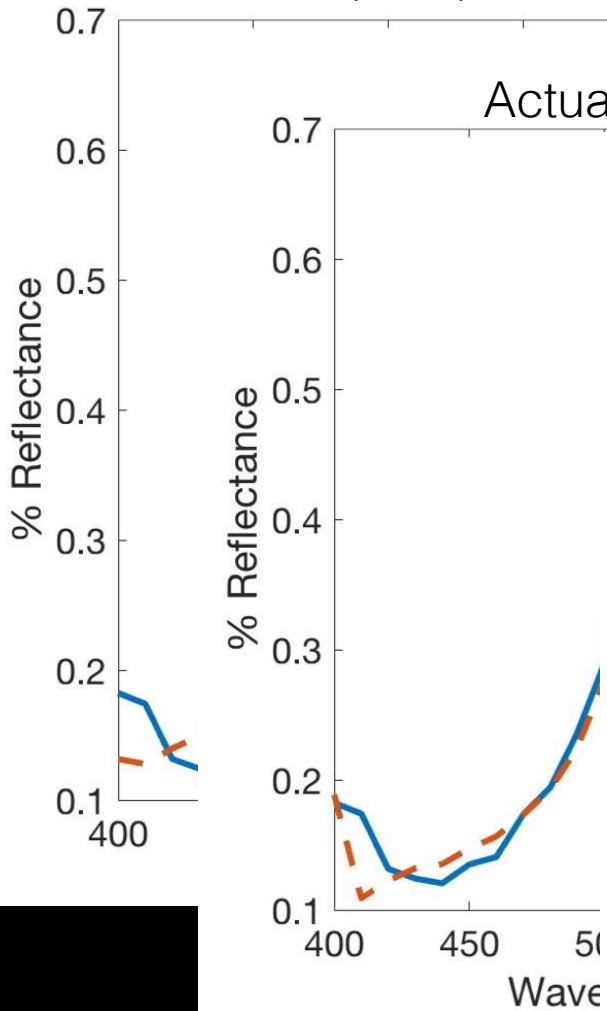


— ground-truth

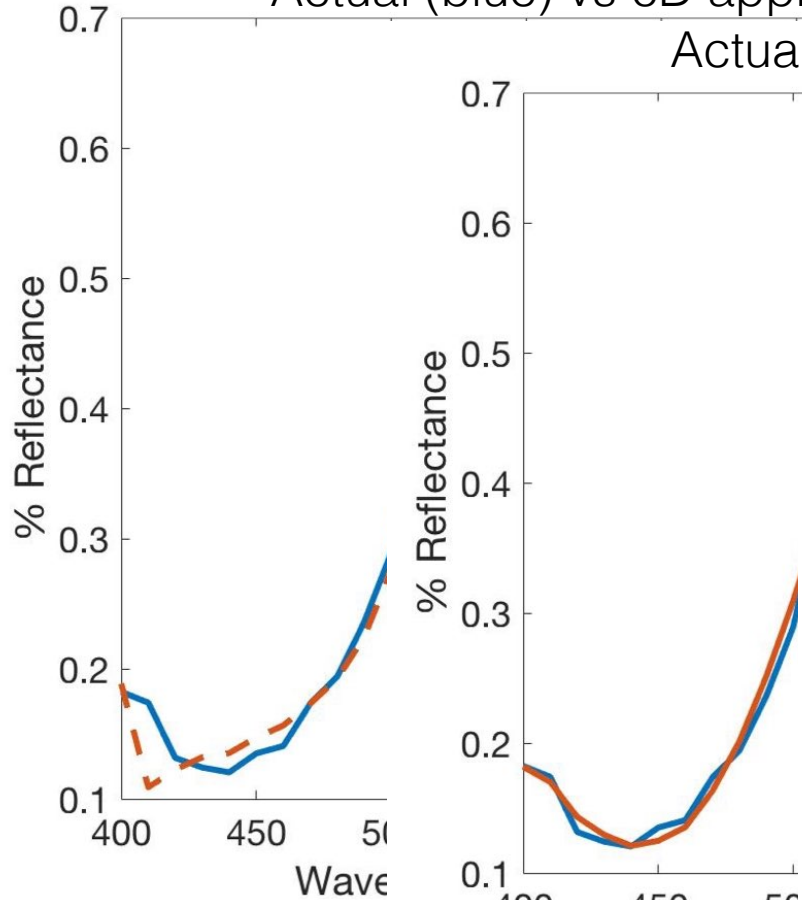
— closest in X-D metamer set

How Many Dimensions?

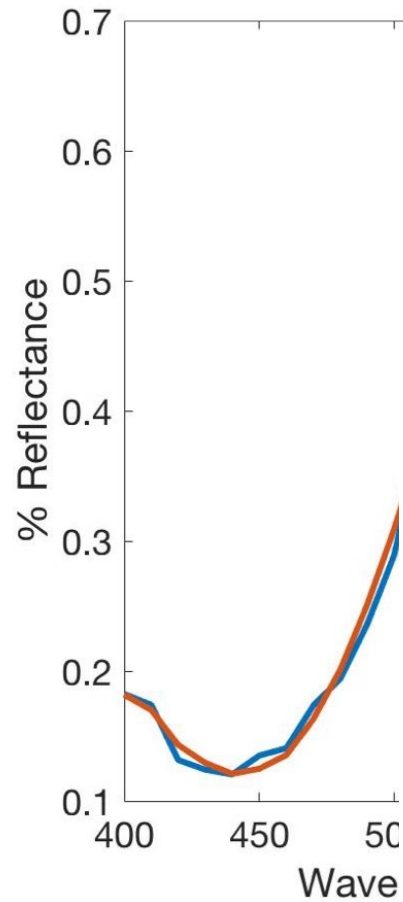
Actual (blue) vs 3D approx



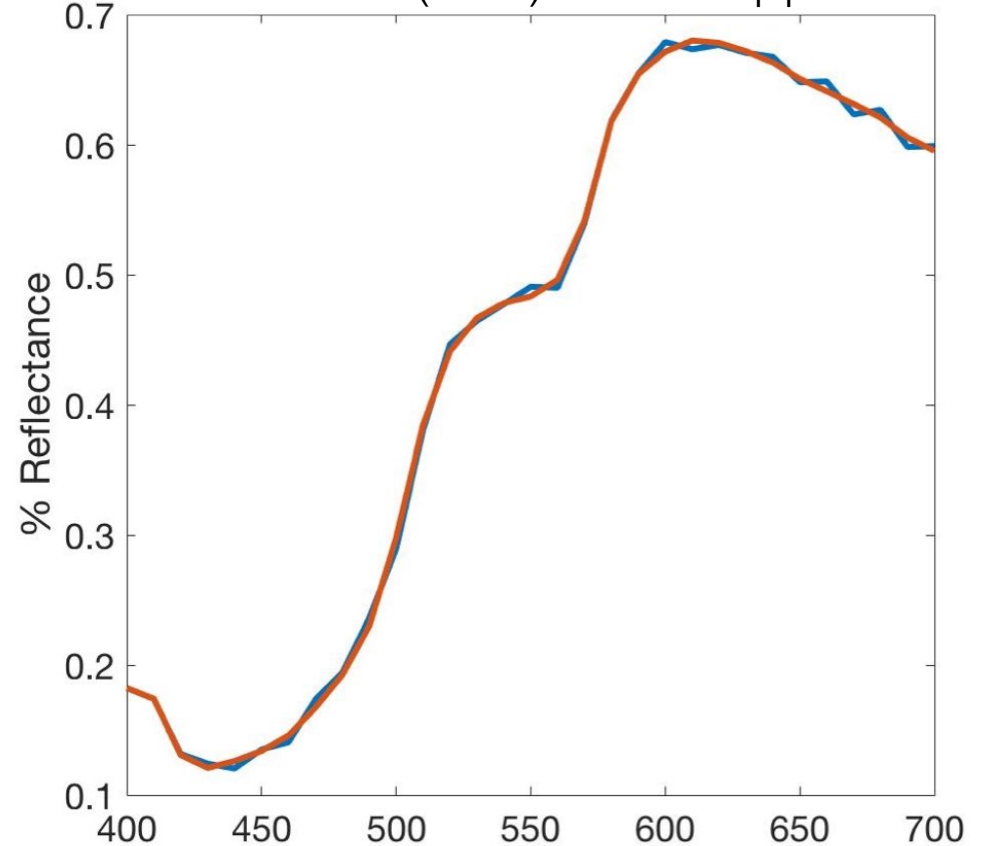
Actual (blue) vs 6D approx



Actual (blue) vs 9D approx



Actual (blue) vs 12D approx



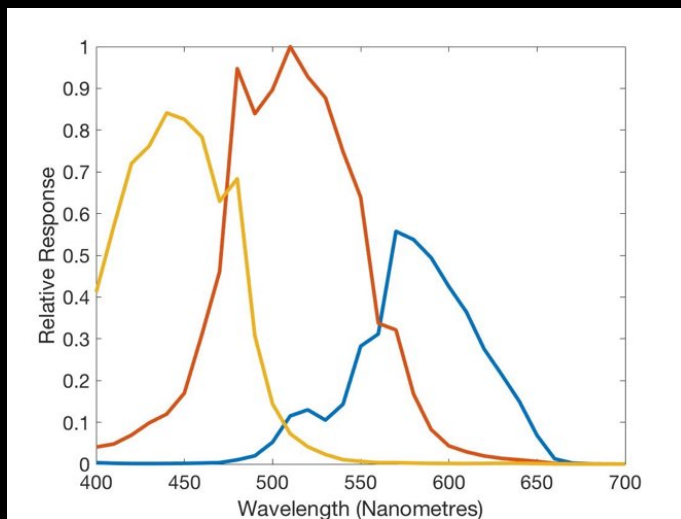
— ground-truth

— closest in X-D metamer set

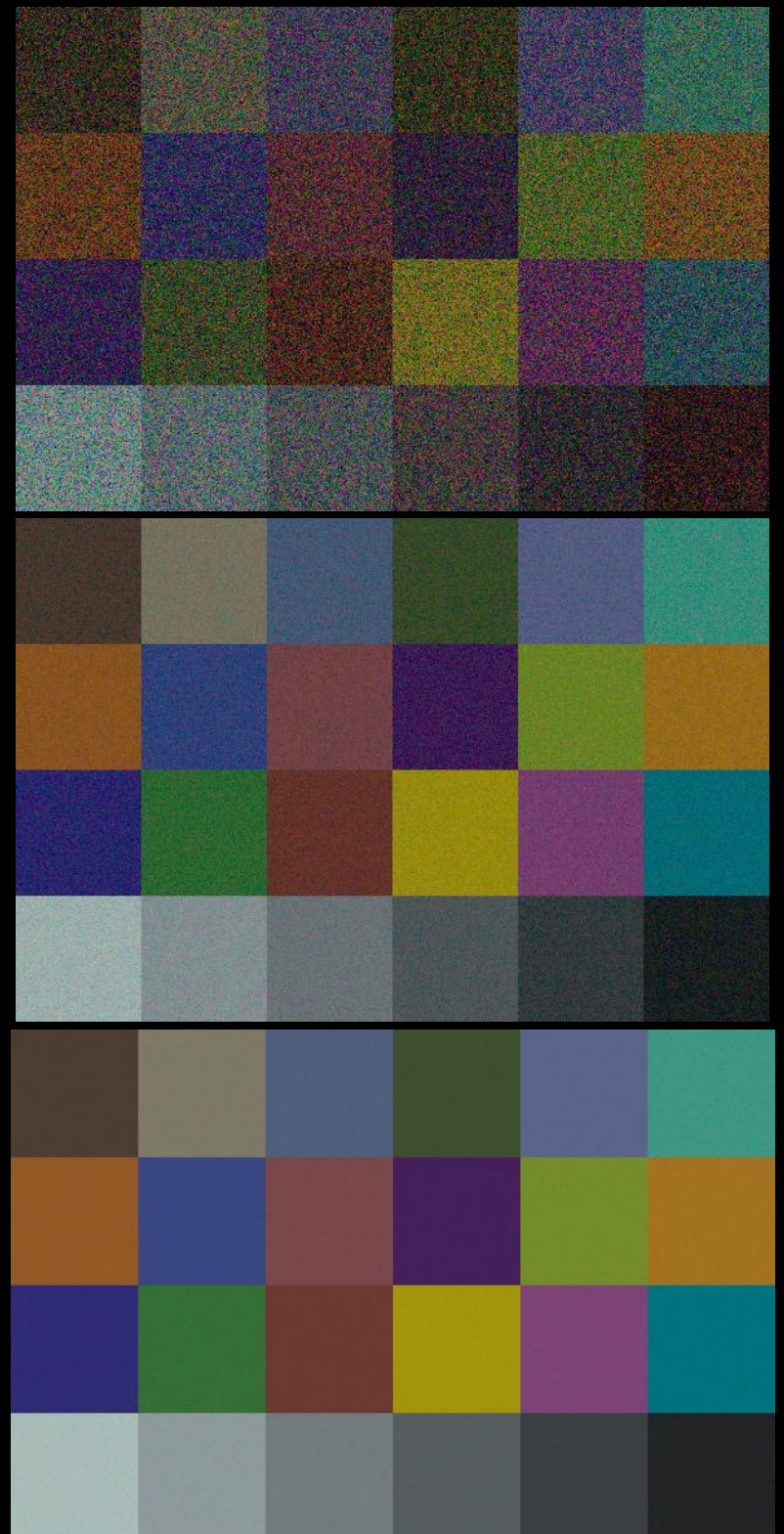
Variants on a theme

Noise: Example

1. Well capacity 50000 electrons
2. Between .1 and 200 photons/10Nm wavelength band
3. 10 bit quantization
4. Sensors scaled so one sensor at one wavelength captures 100% of the photons
5. Shot-noise (assumed normally distributed).

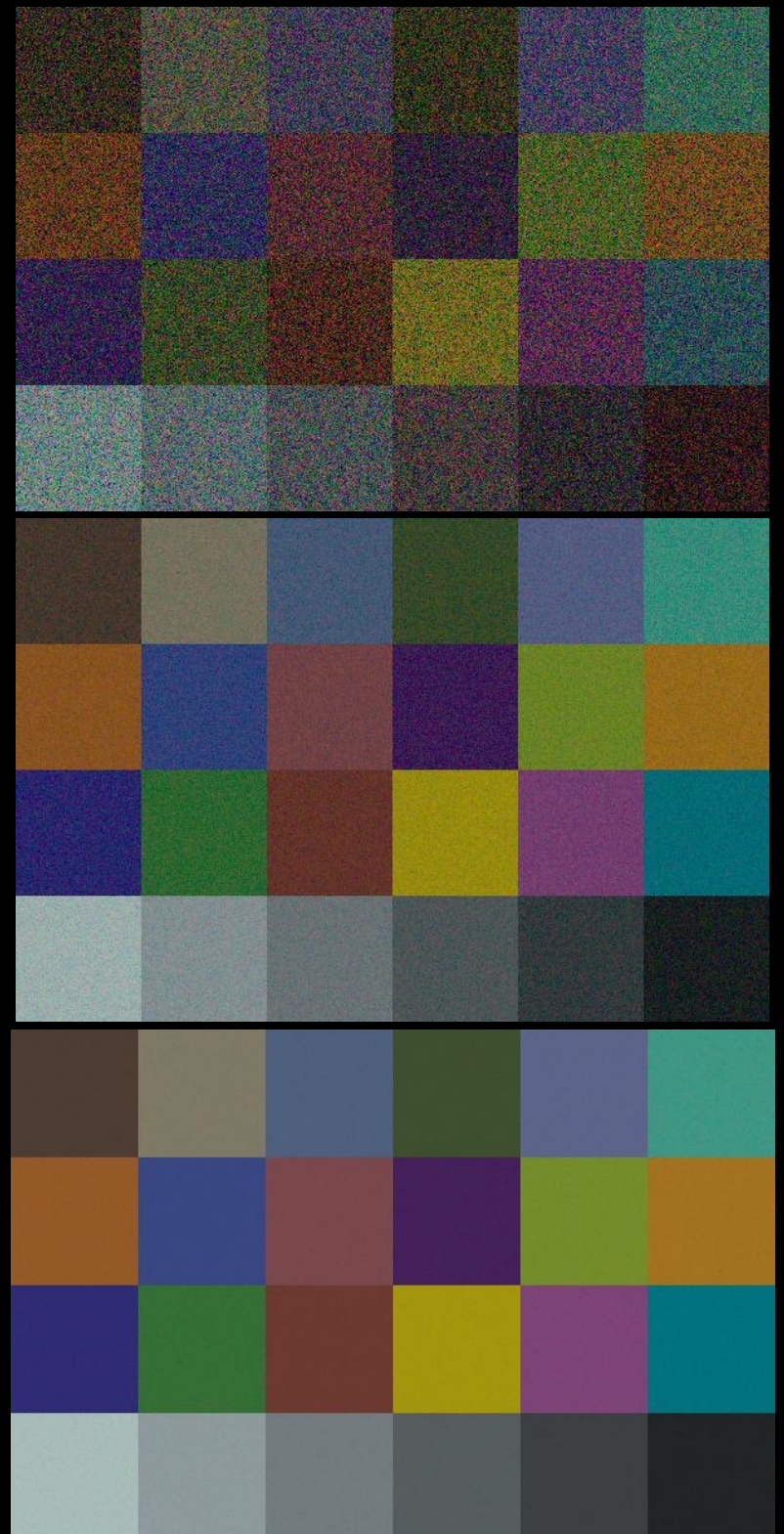


Canon D1
Spectral Sensitivities



What effect does noise
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Intuitively, it becomes larger



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$$\mathcal{R}_k \underline{S} = p_k, \quad k \in \{R, G, B\}$$
$$\underline{0} \leq \underline{S} \leq \underline{1}$$



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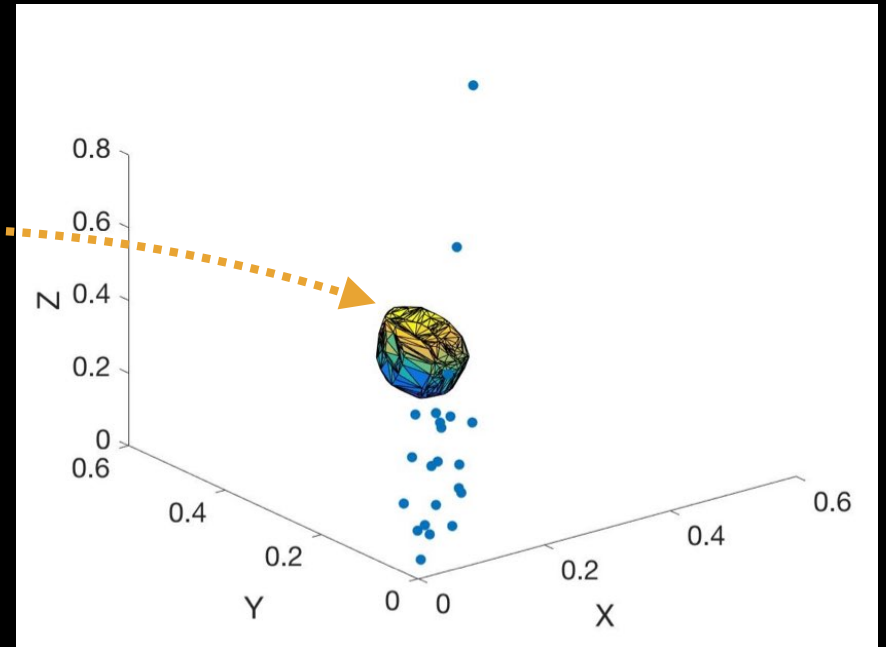
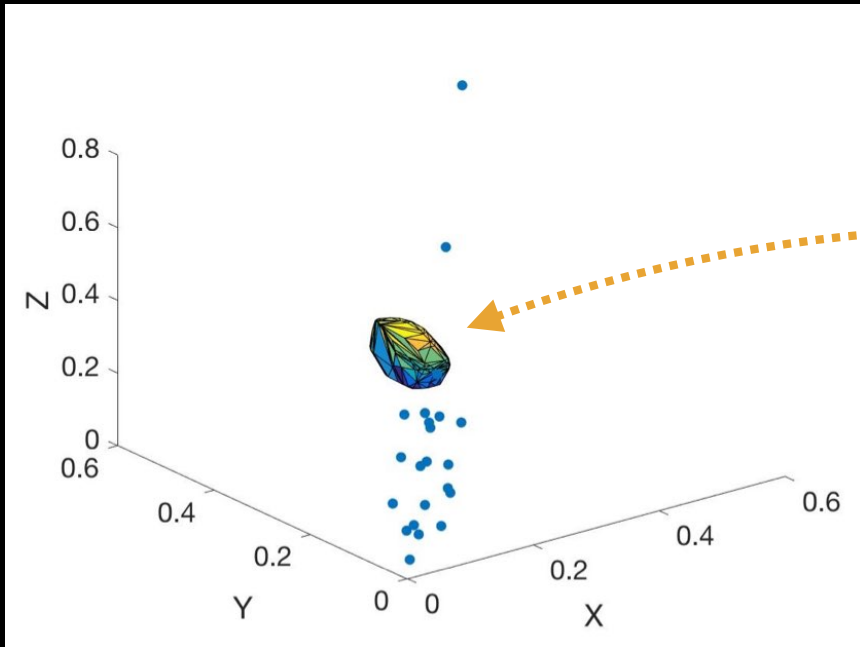
$$\mathcal{R}_k \underline{S} = p_k, \quad k \in \{R, G, B\}$$
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with ϵ noise

$$p_k - \epsilon \leq \mathcal{R}_k \underline{S} \leq p_k + \epsilon, \quad k \in \{R, G, B\}$$
$$\underline{0} \leq \underline{S} \leq \underline{1}$$

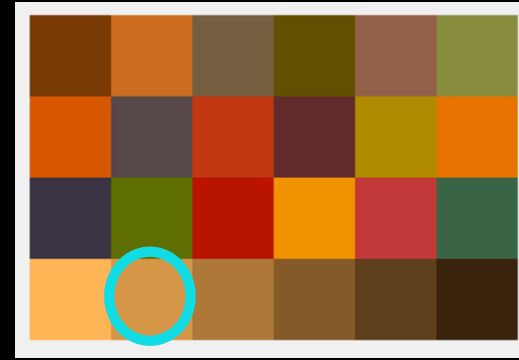
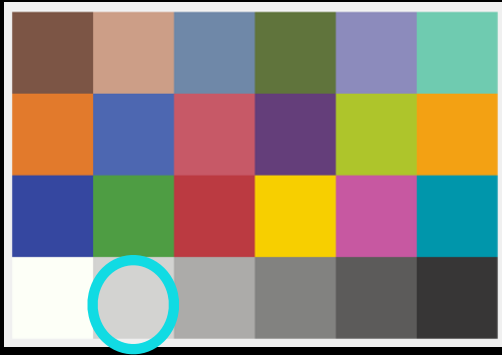


Noise

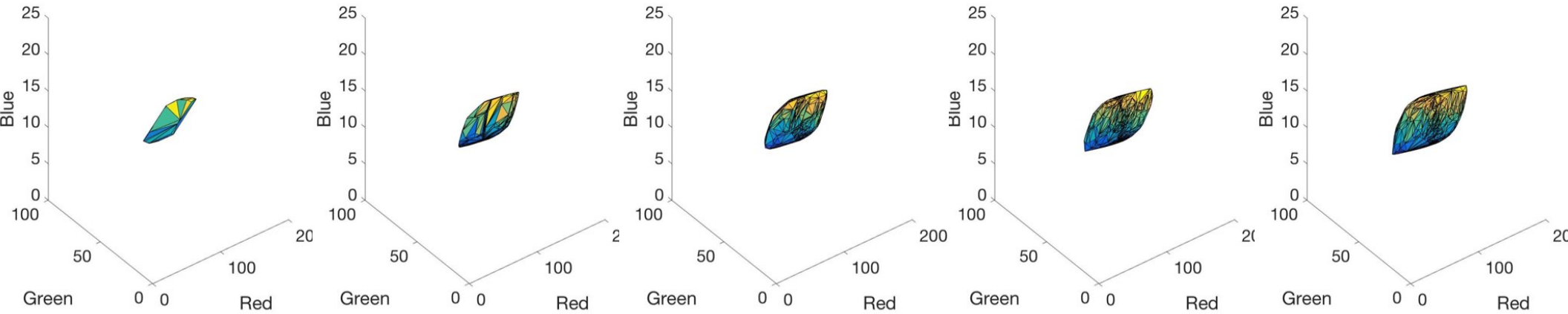


The 'paramer' set (computed with $\sim 2\%$ noise) is 3* the volume of the noise free Metamer set

Noise vs Dimensionality



60% grey that maps to a single RGB could be
the 'projection of many metamers.'
Under a second light the metamer set spans
a set of possible RGBs



6 dimensional
basis

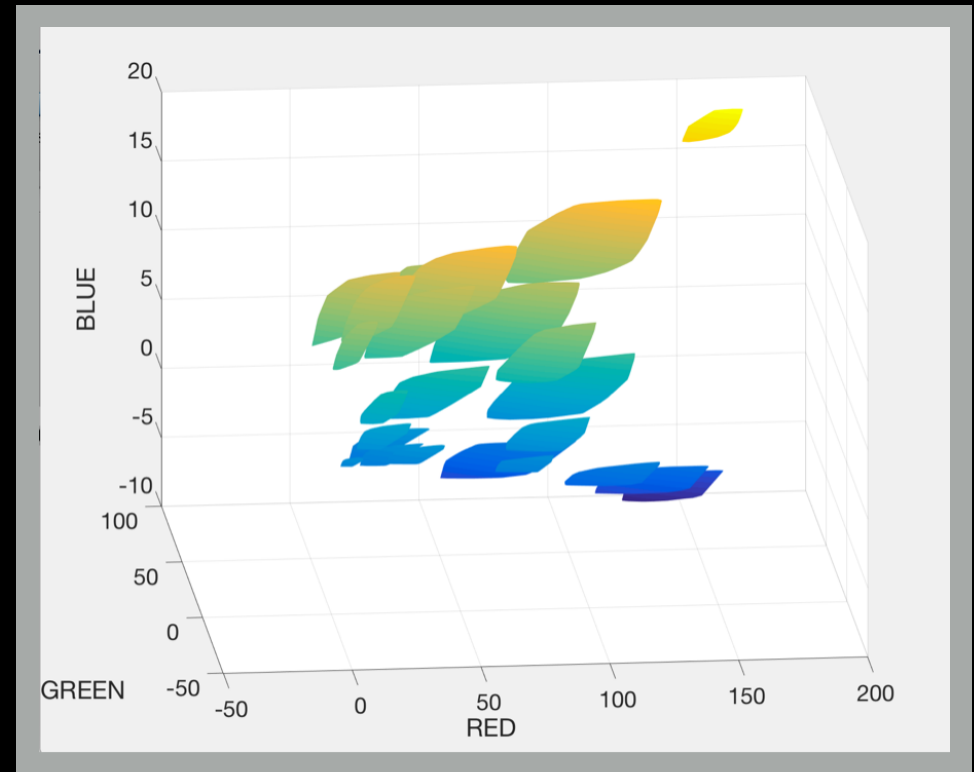
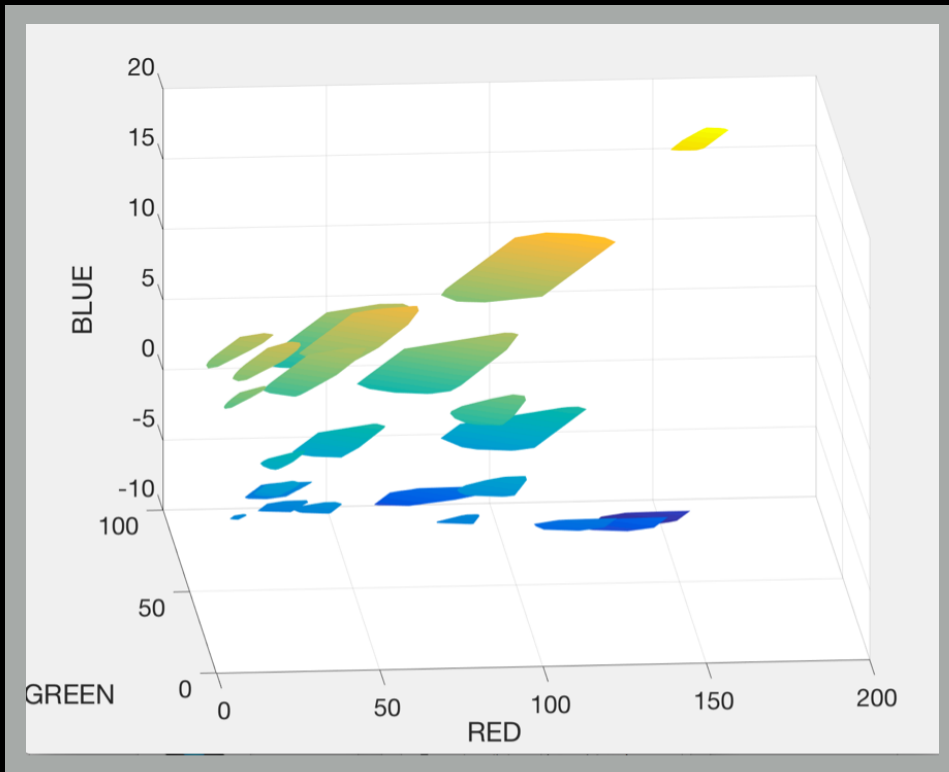
10 dimensional
basis

10 dimensional
basis+noise

'all possible'
spectra

'all possible'
spectra+noise

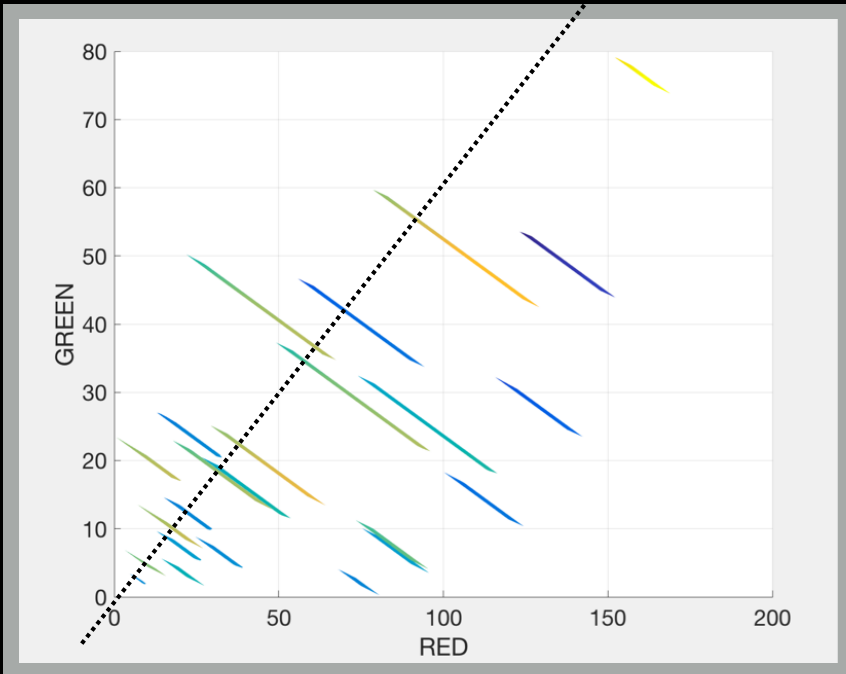
Invariant Metamer Sets



Metamer Set: 6 dimensional basis

Metamer Set: 10 dimensional basis

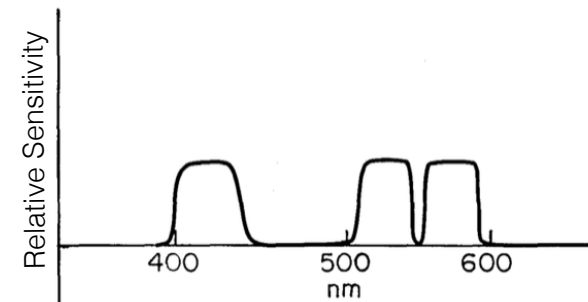
What are good sensors?



i) The projection in the 'luminance' direction has small metamer sets

ii) Possibly, given $N > 3$ sensors, there are linear combinations which give RGB (sRGB?) with small metamer sets (use > 3 sensors to get provably stable 3 sensor measurements)

iii) It can't just be about metamers. Below is an sensor set that has many of the same metamers for all lights



Formal connections between lightness algorithms

Anya Hurlbert

J. Opt. Soc. Am. A/Vol. 3, No. 10/October 1986

Summary

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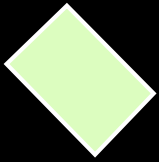


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Summary



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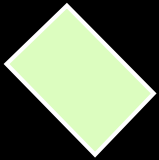


2) Reliably: what the 3 numbers 'mean' and how certain or uncertain they are (are you sure its not a road sign?)

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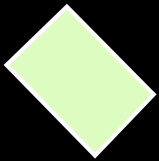


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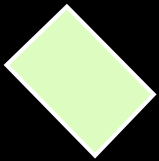


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5) Next Steps? Metamer sets and other tools exist for answering a variety of questions. Example:
"Metamer Constrained Colour Correction"